FM 4–10

3 Jul 194[illegible]

WAR DEPARTMENT

COAST ARTILLERY FIELD MANUAL

SEACOAST ARTILLERY GUNNERY

W/change #1 / April [illegible]

LAWRENCE

COAST ARTILLERY FIELD MANUAL

SEACOAST ARTILLERY

GUNNERY

CHANGES No. 1 } WAR DEPARTMENT,
WASHINGTON, April 24, 1942.

FM 4–10, July 3, 1940, is changed as follows:

■ 22. EXAMPLES.— * * *

* * * * * * *

b. What is the lateral effect of rotation of the earth assuming corrected range to be 25,000 yards?

* * * * * * *

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 24. DETERMINATION OF DIFFERENCE IN ALTITUDE.— * * *

* * * * * * *

Solution:

Apparent difference in altitude (in feet)=3 R (in yards) $\times$ tan ϵ

$$\log 3=0.47712$$
$$\log 7{,}400=3.86923$$
$$\log \tan \epsilon=7.36682-10$$

$$\text{log apparent difference in altitude}=1.71317$$
$$\text{Apparent difference in altitude}=52 \text{ feet.}$$

The actual position of the target is below its apparent position by an amount h, the combined effect of curvature and refraction.

$$h=0.18\times(7.4)^2=10 \text{ feet.}$$

Therefore the target is 52+10 or 62 feet below the guns.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 26. EXAMPLES.

* * * * * * *

c. Assume that a 155-mm gun, using normal charge and firing shell, HE, Mk. III, with fuze, short (Mk. IV*), is emplaced 406 feet above datum level, and is to fire at a destroyer at a range of 6,000 yards from the gun. The tide is * * * to be found. What is the corrected range?

* * * * * * *

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 43. GENERAL.

* * * * * * *

b. After the uncorrected range and azimuth have been determined, the preparation of firing data is completed by applying corrections to these data for all known nonstandard conditions. Uncorrected range is used for computing range corrections and corrected range is used for computing azimuth corrections.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 45. ACCURACY OF COMPUTATIONS.

* * * * * * *

	Solve to or use nearest—
Firing range	10 yards.
Firing elevation	1 mil or minute.
Firing azimuth or deflection	0.01° or 1 mil.
Range for determining differential effects	100 yards.
Range effects, distances in all calculations	1 yard.
Lateral effects	0.01° or 1 mil.
Latitude of gun	1°.
Azimuth of target (rotation or wind)	1° or 1 mil.
Height of site	1 foot.

* * * * * * *

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 47. DEFINITIONS.

* * * * * * *

c. Dead areas.—Areas that cannot be reached by fire. These may be caused by masks in front of the battery as well as by obstructions in the descending path of the projectile and also by height of site of the gun.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 52. GENERAL.—Frequently it is * * * listed in table A of the firing tables. If more accuracy is needed, the problem may be solved by computation from the firing tables, using the method given in paragraph 26*b*.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 53. DETERMINATION.—*a. Approximate solution.*—Using the minimum elevation as determined by one of the methods above, extract from table A of the firing tables the corresponding slope of fall which is given in the general form, 1 on *n*. This determines the slope of the line *BS* in figure 7. Then, by simple proportion, $BB'/B'S=1/n$ and $B'S=n\times BB'$. The approximate range to the splash *S* may then be computed by the formula: Expected range$=GB+B'S$ (fig. 8). Since this method is based on the assumption that the trajectory is a straight line beyond the level point, the approximation will be close only when the angle of fall is large.

*   *   *   *   *   *   *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 56. EXAMPLES.

*   *   *   *   *   *   *

b. Plot of arcs of range from minimum to maximum.

*   *   *   *   *   *   *

(2) *Maximum range.*—From "General Information" * * * yards (table A). From table B we find the map range for a height of target of —200 feet to be approximately 29,370 yards. Arcs for each 2,000 yards of range between the limiting ranges are usually shown on the chart.

*   *   *   *   *   *   *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 69. TEST OF ACCURACY OF AN OBSERVER.

*   *   *   *   *   *   *

Reading No.	True range (by horizontal base)	D. P. F. range	Error D. P. F.	Systematic error	Accidental error
	Yards	*Yards*	*Yards*	*Yards*	*Yards*
* *	*	*	*	*	*
5	10,050	10,020	−30	+18	48
6	9,940	9,950	+10	+18	8
7	9,830	9,880	+50	+18	32
8	9,730	9,730	0	+18	18
9	9,630	9,590	−40	+18	58
10	9,530	9,600	+70	+18	52
Mean	10,000	10,018			34

*   *   *   *   *   *   *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 72. COMPUTATION OF CHECK POINTS, COORDINATES KNOWN.

* * * * * * *

NOTE.—Δy should be corrected for magnification of scale when using standard grid coordinates. (See table XLIX, TM 5–236.)

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 74. EXAMPLES.—*a*. Given the following data (standard grid coordinates):

* * * * * * *

Latitude and longitude of directing point *G*. 37° *N*. and 76°18′ *W*., respectively. Correction to Δy for magnification of scale error=1,06 yards per thousand yards. (See TM 5–236.)

* * * * * * *

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 89. CORRECTIVE MEASURES.—Fixed seacoast guns * * * if required, vertical angles. If a deflection is set on the sight and the gun traversed until the line of sight includes the target, the axis of the trunnions is given a definite direction. Since the axis of the bore * * * "compensating sight mounts."

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 102. DEVIATIONS.

* * * * * * *

b. The *center of impact*, or mean point of impact, of a series of shots is a point whose position is fixed by the positions of the several points of impact. The range deviation of the center of impact is the algebraic mean of the range deviations of the separate impacts.

* * * * * * *

■ 119. EXAMPLES.

* * * * * * *

d. * * *

* * * * * * *

For tactical No. 4, register No. 47:

$$\frac{(3\times8\times(+8))+(8\times(+16))+(8\times(+10))+(10\times(+14))+(8\times(+10))}{(3\times8)+8+8+10+8}=\frac{+620}{58}=+11 \text{ f/s}$$

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 128. COMPUTATION OF PROBABILITY IN OTHER OPERATIONS.—The method of calculating the probability of a shot's falling between certain points given in paragraph 127 is equally applicable to the calculation of the probability that any variable distributed in the same manner will take on a value between specified limits. It has been mentioned in paragraph 107 that accidental errors are usually considered as distributed in this way. In fact, the study of the distribution of accidental errors * * * calibration, and pointing.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 130. COMPOUND ERRORS.

* * * * * * *

b. The spotting error is not independent of the magnitude of the deviation (the larger the deviation the less accurate the spot), so that in compounding it with other errors the second of the conditions listed in the rule above is not fulfilled. It is permissible to assume * * * error of observation.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 134. BASIC PRINCIPLES.— * * *

* * * * * * *

j. Having decided that a correction is necessary, it should be made to the nearest 1/10 of 1 percent of the range or nearest 10 yards.

* * * * * * *

r. Occasionally an erratic or wild shot will be fired. A shot should be considered wild when its impact is more than four developed armament probable errors or, in the absence of this information, more than six firing table probable errors from the center of impact. A wild shot should be disregarded in determining an adjustment correction. Obviously, a wild shot cannot be identified until sufficient rounds have been fired to give a reasonably accurate location of the center of impact.

* * * * * * *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 139. MAGNITUDE METHOD (DEVIATIONS MEASURED).—In this method of adjustment of fire, the magnitude and sense of the range deviation (in terms of its corresponding correction) of each shot or salvo center of impact are spotted and the impacts are plotted graphically on the fire adjustment board. (See FM 4-15.) Corrections, mathematically as correct * * * slow rate of fire.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 142. ADJUSTMENT FOR DIRECTION.

* * * * * * *

c. When employing case II fire, lateral corrections may be made by an axial observer located near the guns who calls the ~~correction~~ deflection.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 143. GENERAL.—In this method, the magnitude and the sense of the range deviation (in terms of its corresponding correction) of the center of impact of a series of shots or salvos are the basis for determining the range correction to be applied. On the fire adjustment board * * * on the same correction.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 146. EXAMPLES.—*a*. The following examples of range adjustment are based on the use of the fire adjustment board (see FM 4-15). The standard system of reference numbers is used in which 300 represents a zero correction, and the digit in the units' place represents tenths of 1 percent. For example, 315 represents a correction of up 1.5 percent. The data for the examples were determined by means of the dispersion tape and scale described in appendix I. A probable error of 1 percent is assumed for convenience in all examples.

b. In the examples, certain conventions have been followed as indicated below:

(1) A cross (X) is used to denote a single shot. (A cross with an exponent would be used to denote the center of impact of a salvo, the exponent being the number of shots in the salvo.)

* * * * * * *

(3) A check mark is used to show two things, the first being the location of the center of impact of the shots considered as a basis for a correction, and the second being the magnitude of the adjustment correction.

(4) The numbers immediately above a check mark indicate, in reference numbers, the correction ordered.

* * * * * * *

(6) The vertical scale is uniform. A different horizontal line is used for each salvo both in trial fire and fire for effect. When conducting trial fire by single shots, if a correction

is applied after the first shot, then the second shot is plotted alone on the next line and the succeeding shots of trial fire are plotted two to a line. During fire for effect, the shots are plotted two to a line. The board presents at all times a chronological record of the fire adjustment. ASSUME BELL FIRING

* * * * * * *

c. For the examples in this section, the assumed situation is as follows:

* * * * * * *

Example No. 1.—Tabulated data.

Shot No.	Uncorrected range plus ballistic correction (yards)	Adjustment correction		Point of impact		Corrected range (yards)
		Percent	Yards	Percent	Yards	
T. S. 1	12,050	300	0	**258**	+510	12,050
T. S. 2	12,010	258	−500	**320**	−240	11,510
T. S. 3	11,970	258	−500	**315**	−180	11,470
T. S. 4	11,930	258	−500	**304**	−50	11,430
S–1	11,640	268	−370	**320**	−230	11,270
	11,640	268	−370	**290**	+120	11,270
S–2	11,530	268	−370	**318**	−210	11,160
	11,530	268	−370	**314**	−160	11,160
S–3	11,420	268	−370	**297**	+30	11,050
	11,420	268	−370	**325**	−290	11,050
S–4	11,300	273	−310	**300**	0	10,990
	11,300	273	−310	**308**	−90	10,990
S–5	11,180	273	−300	**302**	−20	10,880
	11,180	273	−300	**294**	+70	10,880
S–6	11,060	278	−240	**307**	−80	10,820
	11,060	278	−240	**291**	+100	10,820
S–7	10,940	278	−240	**301**	−10	10,700
	10,940	278	−240	**295**	+50	10,700

In example No. 1 (fig. 36), the first ranging shot was reported as 258 or over 510 yards. Since this shot was more than three probable errors from the target, a correction of 258 was ordered to bring the remaining trial shots closer to the target. The deviations of all four trial shots were considered * * * from the line of targets, and no correction was applied.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

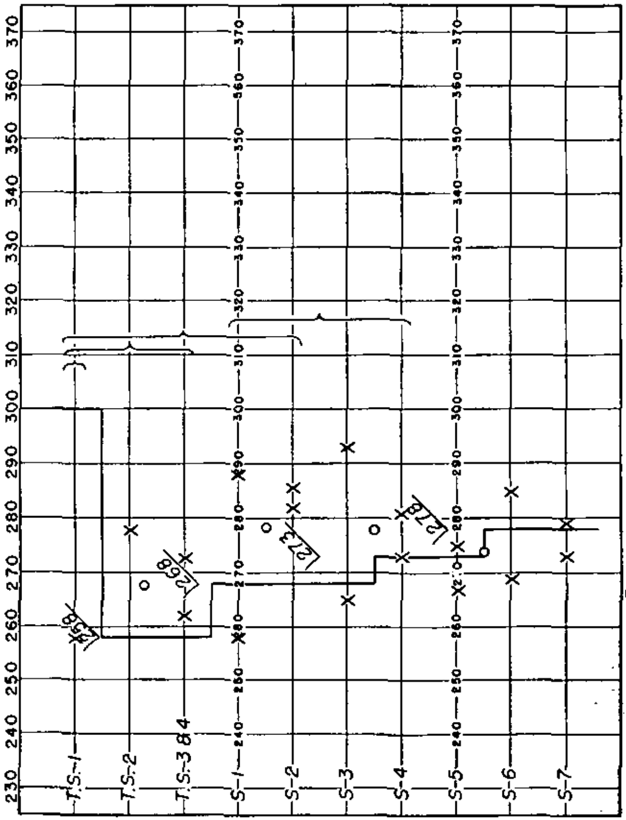

FIGURE 36.—Adjustment of fire, magnitude method (example No. 1).

Example No. 2.—Tabulated data.

Shot No.	Uncorrected range plus ballistic correction (yards)	Adjustment correction		Point of impact		Corrected range (yards)
		Percent	Yards	Percent	Yards	
T. S. 1	10, 100	300	0	300	0	10, 100
T. S. 2	10, 400	300	0	324	−250	10, 400
T. S. 3	10, 730	300	0	284	+170	10, 730
T. S. 4	11, 030	300	0	330	−330	11, 030
S-1	11, 560	310	+120	316	−180	11, 680
	11, 560	310	+120	318	−210	11, 680
S-2	11, 760	310	+120	297	+40	11, 880
	11, 760	310	+120	314	−160	11, 880
S-3	11, 980	310	+120	305	−60	12, 100
	11, 980	310	+120	314	−170	12, 100
S-4	12, 180	310	+120	300	0	12, 300
	12, 180	310	+120	312	−150	12, 300
S-5	12, 390	310	+120	322	−270	12, 510
	12, 390	310	+120	314	−170	12, 510
S-6	12, 590	319	+240	308	−100	12, 830
	12, 590	319	+240	282	+230	12, 830

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

In example No. 2 (fig. 37), * * * and no correction was ordered. However, the center of impact of the second series of four shots of record fire combined with that of the first series of four shots of record fire indicated a correction of 319. This correction was ordered, taking effect on salvo No. 6 of fire for effect.

Example No. 3.—Tabulated data.

* * * * * * *

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 150. Examples.— * * *

* * * * * * *

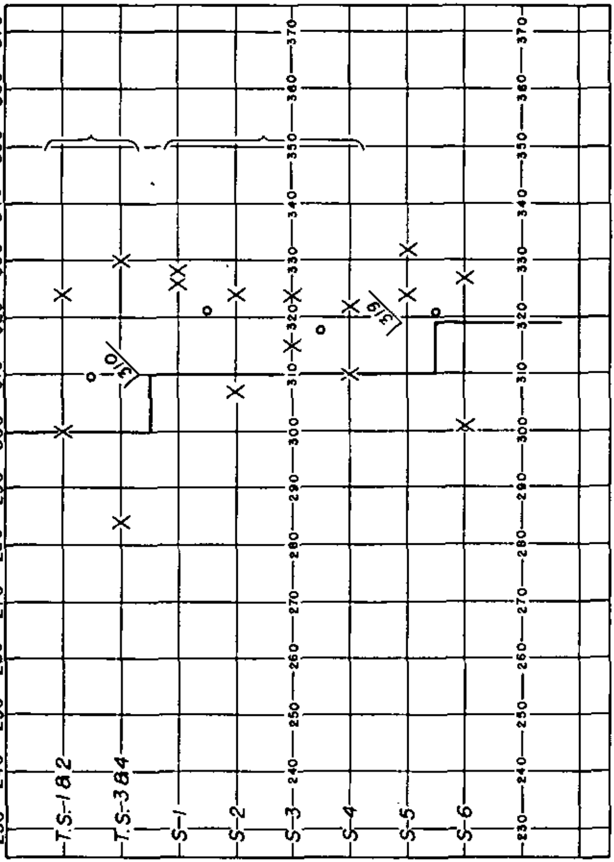

FIGURE 37.—Adjustment of fire, magnitude method (example No. 2).

a. Example No. 1.—A battery of four 155-mm guns using normal charge was fired at a target according to the following tabulation ;

Shot No.	Range sensings	Adjustment correction		Uncorrected range plus ballistic correction (yards)	Corrected range (yards)
		Percent	Yards		
T. S. 1	O-O-O-O	300	0	10, 100	10, 100
T. S. 2	S-S-S-S	272	−290	10, 200	9, 910
1	S-H-S-S	286	−150	10, 300	10, 150
* *	*	*	*	*	*

* * * * * * *

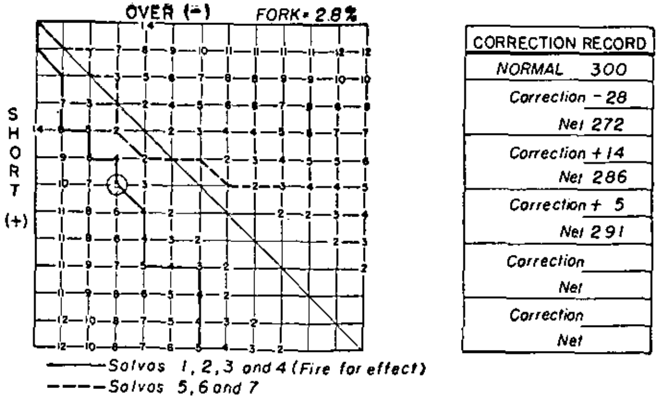

FIGURE 39.—Adjustment of fire, bracketing method (example No. 1).

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

* * * * * * *

c. Example No. 3.—The following tabulation shows the data of a firing by the same battery of 155-mm guns.

* * * * * * *

Figure 41 shows the adjustment of fire in this practice. The first trial salvo was short, causing a correction of up one fork, or 2.8 percent, to be applied. The second trial salvo produced a hit, two overs, and one short, and fire for effect was begun with this same adjustment. The impacts of the second trial salvo were plotted on the chart to be considered with the first impacts of fire for effect. After salvos Nos. 1 and 2 of fire for effect had been plotted, no correction was found necessary. However, at this time it was noted that the center of impact of salvo No. 2 was definitely over, and it was decided to observe the next salvo carefully to see if its center of impact also was over. (A new line of impacts was started on the chart with salvo 3 because a correction should not be based on more than 12 impacts and one more salvo on the first line of impacts would have made 16 impacts.) Salvo No. 3 was spotted as three overs and a short, confirming the suspicion that a down correction was in order. In making a correction at this point * * * on the old adjustment were plotted as usual.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 153. GLOSSARY.

* * * * * * *

Predicting interval.—The interval between successive predictions of future positions of the target.

* * * * * * *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

APPENDIX I

DISPERSION TAPE AND SCALE FOR USE IN FIRE ADJUSTMENT PROBLEMS WITH SIMULATED FIRE

■ 4. DEVIATION SCALE (fig. 2).—*a. General.*

* * * * * * *

(2) An auxiliary deviation scale marked "over," "short," and "hit" is provided for use with the bracketing method of adjustment. The width of the space marked "hit" on the deviation scale may be determined from the size of the danger space of the average target at medium range.

* * * * * * *

c. Operation in drill.—Rescinded.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

■ 5. Fixed Scale.—This scale is graduated to the same scale and marked with the same reference numbers as the deviation scale. It is fixed to the mount just below the deviation

CORRECTION RECORD	
NORMAL	300
Correction	+28
Net	328
Correction	− 8
Net	320
Correction	
Net	
Correction	
Net	
Correction	
Net	

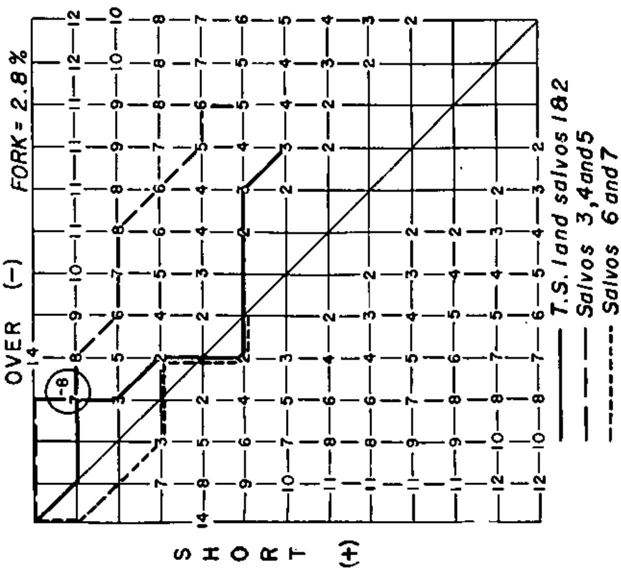

FIGURE 41.—Adjustment of fire, bracketing method (example No. 3).

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

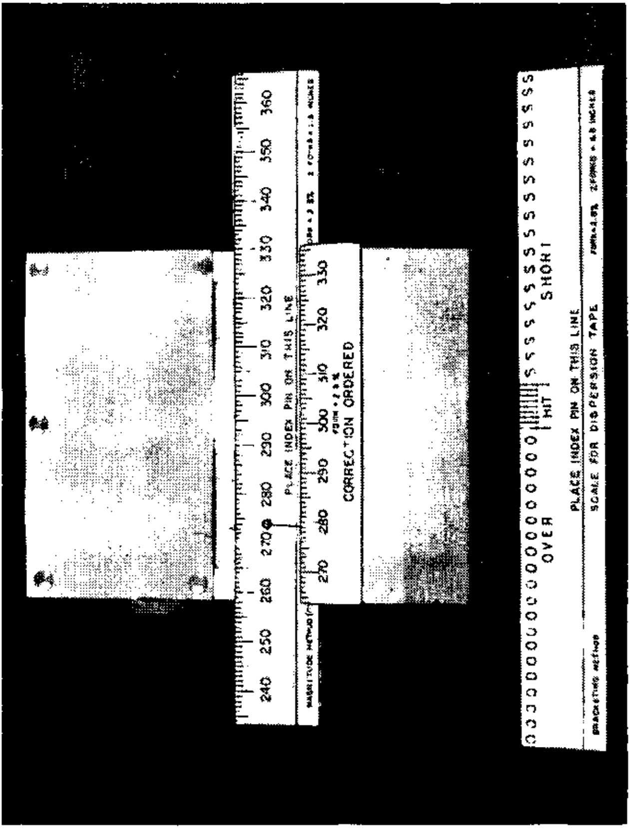

FIGURE 2.—Dispersion tape and scale.

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

scale so that its normal (300) is on line with the center line of the dispersion line of the dispersion tape.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

■ 6. Operation of Dispersion Tape in Drill.—*a*. Place the deviation scale in position under the window, displacing the normal (300) the desired distance from the center of dispersion. Place pin on deviation scale opposite normal (300) on fixed scale.

b. Determine a rule to be followed in selecting deviations and, following that rule, bring the proper frame on the dispersion tape into view in the window.

c. At the proper time, read the deviation from the deviation scale opposite the mark that represents the splash. Set the tape for the next reading according to the predetermined rule.

d. When an adjustment is ordered, move the deviation scale until the pin is opposite the correction ordered on the fixed scale. This move must be timed to synchronize with the fall of the shot on which the correction is applied.

e. Do not move the pin until the problem is completed unless it is desired to simulate a shifting center of dispersion. If such action is desired, shift the pin in the amount and direction desired.

[A. G. 062.11 (2–7–42).] (C 1, April 24, 1942.)

APPENDIX IV

PRINCIPLES OF VERTICAL BASE POSITION FINDING

■ 6. EXAMPLE.— * * *

* * * * * * *

Term	Logarithm	Natural number	
$\tan \alpha$	7.63982−10		0.0043633
$\tan^2 \alpha$	5.27964−10	0.000019039	
4C	3.39129−10		
b	1.60206		
$4Cb$	4.99335−10	0.000009848	
$\tan^2 \alpha - 4Cb$	4.96336−10	0.000009191	
$\sqrt{\tan^2 \alpha - 4Cb}$	7.48168−10		0.0030317
Numerator	7.12437−10		0.0013316
Denominator ($2C$)	3.09026−10		
R	4.03411	10,817 yards.	

* * * * * * *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

APPENDIX VI

TABLES

TABLE I.—*Vertical effect of curvature and refraction* (par. 58*c*)

* * * * * * *

[A. G. 062.11 (2-7-42).] (C 1, April 24, 1942.)

BY ORDER OF THE SECRETARY OF WAR:

G. C. MARSHALL,
Chief of Staff.

OFFICIAL:

J. A. ULIO,
Major General,
The Adjutant General.

U. S. GOVERNMENT PRINTING OFFICE: 1942

FM 4-10

COAST ARTILLERY FIELD MANUAL

SEACOAST ARTILLERY GUNNERY

Prepared under direction of the
Chief of Coast Artillery

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1940

For sale by the Superintendent of Documents, Washington, D. C. - Price 25 cents

WAR DEPARTMENT,
WASHINGTON, *July 3, 1940.*

FM 4–10, Coast Artillery Field Manual, Seacoast Artillery, Gunnery, is published for the information and guidance of all concerned.

[A. G. 062.11 (4–30–40).]

BY ORDER OF THE SECRETARY OF WAR:

G. C. MARSHALL,
Chief of Staff.

OFFICIAL:
E. S. ADAMS,
Major General,
The Adjutant General.

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TABLE OF CONTENTS

COAST ARTILLERY FIELD MANUAL

SEACOAST ARTILLERY

GUNNERY

(The matter contained herein supersedes Chapter 1, Part Two, Coast Artillery Field Manual, Volume I, February 1, 1933; and TM 2160–30, July 10, 1937.)

CHAPTER 1

GENERAL

■ 1. PURPOSE AND SCOPE.—*a*. The purpose of this manual is to provide a compilation of the basic principles underlying the practice of gunnery for officers conducting the fire of seacoast artillery batteries. It is intended as a textbook for the study of gunnery by those preparing for the duties of battery officers and a reference book for those engaged in the training of seacoast artillery batteries.

b. This manual covers the more essential theoretical principles which the battery commander must apply in order to conduct accurate fire. Reference to instruments used to facilitate these operations is made only to illustrate the principles discussed.

■ 2. DEFINITION OF GUNNERY.—Gunnery has been defined as the science and art of firing guns. It includes a study of the flight of the projectile and of the technical considerations involved in the conduct of fire. In order to conduct the fire of his battery with maximum effect, the battery commander must have a thorough working knowledge of the characteristics of his weapon and its ammunition, of the factors that influence the flight of a projectile, of the methods of determining data with which to point the guns, and of the observation and adjustment of fire to improve its accuracy. Proper coordination and use of this knowledge in the training of the personnel of his organization will enable him to employ his weapons to the maximum advantage.

■ 3. REFERENCES.—For detailed description of the important features of design and operation of the instruments referred to in paragraph 1, see FM 4–15 and pertinent Technical Manuals. In addition, valuable reference matter may be found in the following publications:

Text on Exterior Ballistics, the Ordnance School, Ordnance Department, United States Army.

Computation of Firing Tables for United States Army, H. P. Hitchcock.

Elements of Ordnance, Hayes.

Ordnance and Gunnery, McFarland.

Naval Ordnance, United States Naval Institute.

CHAPTER 2

ELEMENTS OF BALLISTICS

SECTION I

GENERAL

■ 4. GENERAL.—Ballistics is the science that treats of the motion of projectiles. It is the theoretical foundation on which must be based all improvements in the design of guns and ammunition leading to the increased power and efficiency of artillery. Ballistics is divided into two main branches; interior ballistics and exterior ballistics.

■ 5. INTERIOR BALLISTICS.—Interior ballistics is the study of the motion of a projectile while still in the bore of the cannon. Its principal object is to determine the relations which connect the weight of the projectile, weight and other characteristics of the powder, and dimensions of the cannon with the velocity of the projectile at any point in the bore and the accompanying powder gas pressures. It is of use principally in designing new weapons. The practical artilleryman is, however, interested in some parts of this subject, such as the muzzle velocity, maximum pressure, and factors governing erosion of the bore of the cannon.

■ 6. EXTERIOR BALLISTICS.—Exterior ballistics treats of the motion of a projectile after it has left the bore, including both the projectile in flight and the factors affecting its flight. It is of special importance to the artilleryman. It has practical application in the computation of firing tables and in the determination of corrections to be applied to the firing data to offset the effect of wind, air density, and other measurable factors on the projectile.

SECTION II

TRAJECTORY AND ITS ELEMENTS

■ 7. GENERAL.—Trajectory is the path followed by the projectile from the muzzle of the gun to the point where it strikes.

The phrase "elements of the trajectory" is applied to the various features of the trajectory (fig. 1); they are defined in the paragraphs below.

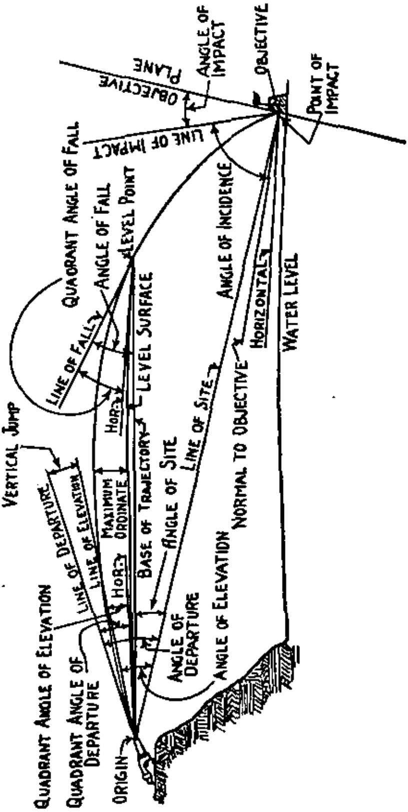


FIGURE 1.—Elements of trajectory in air.

■ 8. INTRINSIC ELEMENTS.—*a*. *Trajectory* is the curve described by the center of gravity of a projectile in flight.

b. *Ascending branch* is that portion of the trajectory described by the projectile while going up.

c. *Descending branch* is that portion of the trajectory described by the projectile while coming down.

d. *Origin* is the center of the muzzle of the piece at instant of departure.

e. *Summit* is the highest point on the trajectory.

f. *Level point* is the point on the descending branch of the trajectory which is at the same altitude as the origin. It is also called *point of fall.*

g. *Base of the trajectory* is the straight line joining the origin and the level point.

h. *Maximum ordinate* is the difference in altitude between the origin and the summit.

■ 9. INITIAL ELEMENTS.—*a. Line of elevation* is the axis of the bore prolonged when the piece is laid.

b. *Line of departure* is the axis of the bore prolonged when the piece is fired. It is tangent to the trajectory at its origin.

c. *Plane of fire* is the vertical plane containing the line of elevation.

d. *Plane of departure* is the vertical plane containing the line of departure.

e. *Vertical jump* is the difference between the angle of elevation and the angle of departure. It is positive if the angle of departure is greater than the angle of elevation.

f. *Lateral jump* is the horizontal angle between the plane of fire and the plane of departure.

g. *Line of site* is the straight line joining the origin and the target.

h. *Angle of site* (ϵ) is the angle between the line of site and base of the trajectory.

i. *Angle of elevation* or *elevation* is the angle between the line of elevation and line of site.

j. *Quadrant angle of elevation* (ϕ) or *quadrant elevation* is the angle between the line of elevation and the horizontal.

k. *Angle of departure* is the angle between the line of departure and line of site.

l. *Quadrant angle of departure* (ϕ') is the acute angle between the line of departure and the horizontal.

■ 10. TERMINAL ELEMENTS.—*a. Point of impact* is the point where the projectile first strikes the ground or other material object. It is also called *objective point.*

b. Objective plane is the plane tangent to the surface of the target at point of impact.

c. Line of fall is the tangent to the trajectory at level point.

d. Line of impact is the tangent to the trajectory at point of impact.

e. Angle of impact is the acute angle between the objective plane and line of impact.

f. Angle of incidence is the acute angle between the line of impact and the normal to objective plane at point of impact.

g. Angle of fall (ω) is the angle between the line of fall and base of trajectory.

h. Quadrant angle of fall (ω') is the acute angle between line of fall and the horizontal.

■ 11. OTHER ELEMENTS.—*a. Muzzle velocity or initial velocity* (M.V. or V.) is the velocity with which the projectile is assumed to leave the muzzle of the gun. It is the velocity of the projectile, measured at a distance from the muzzle, corrected for the theoretical loss in velocity during the travel from the origin of the trajectory to point of measurement, considering that during that travel the projectile has been acted upon only by air resistance and gravity.

b. Remaining velocity at any point on the trajectory is the actual velocity along the trajectory at that point.

c. Terminal velocity ($V\omega$) is the remaining velocity at the level point.

d. Time of flight (t) is the time from the instant of departure to the instant that the projectile reaches the point of impact.

e. Range is the distance from the gun or directing point, measured along a great circle of a sphere, concentric with the surface of the earth and passing through the gun or directing point to the target or vertical projection of the target on that sphere. Ranges measured by the standard position-finding systems are not curved ranges, but the error made in assuming that they are curved ranges is negligible for all distances involved in artillery firing.

f. Drift is the divergence of a projectile from the plane of departure due to rotation of the projectile and resistance of the air. It may be expressed either in linear or angular units.

■ 12. TRAJECTORY IN VACUO.—One of the major forces acting on a projectile in flight is gravity. Assume that a projectile is fired in vacuo with a velocity at the muzzle of the gun of V feet per second in the direction OM, as shown in figure 2, and at a vertical angle ϕ' from the horizontal. Assume, in addition, that the force of gravity is constant and acts at right angles to the base of the trajectory throughout the flight of the projectile. During its flight, the projectile is acted upon only by gravity and a study of the resultant trajectory reveals the following facts:

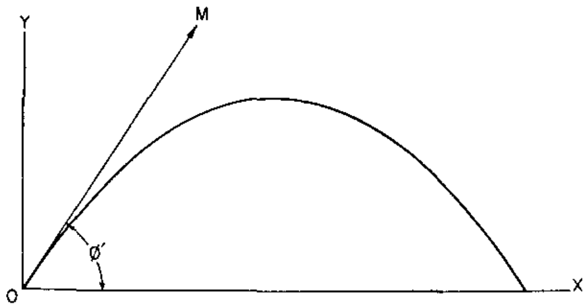


FIGURE 2.—Trajectory in vacuo.

a. The trajectory is a parabola.

b. The trajectory is symmetrical in respect to the maximum ordinate; the ascending and descending branches are the same length and are traversed in the same time, and the quadrant angle of fall is the same as the quadrant angle of departure.

c. The trajectory depends on the initial velocity V and the quadrant angle of departure ϕ' only; the shape of the trajectory is independent of the shape and weight of the projectile.

d. Terminal velocity is the same as initial velocity.

e. Maximum range is attained at a quadrant angle of departure of 45°.

f. The trajectory lies in the plane of departure.

■ 13. AIR RESISTANCE.—It is obvious that for ballistic purposes the air resistance to a moving body is not, like gravity, a constant force, but that it increases with the speed of the body.

Before the nature of the trajectory in air can be studied, it is necessary to determine by experiment the manner in which the resistance encountered by the projectile varies with the speed.

a. The first suggestions as to the laws governing the resistance of bodies moving through the air were advanced by Newton. His theories were that the air resistance is proportional to density of the air, area of cross section of the body, and square of the velocity. When the law based on these theories is tested by experiment with a projectile it is found that the first and second are very accurately verified but that the third, although true within certain limits of velocity, is not even approximately correct for velocities of the projectile outside of these limits. Experiments have been carried on from Newton's day to the present to establish the correct law.

b. There are many things that complicate the determination of the air resistance and its resulting reaction on a projectile. When a projectile is fired from a cannon, it acquires a certain amount of kinetic energy. In overcoming air resistance, part of this energy is used up. This loss of energy may be accounted for mostly as follows: displacing a certain volume of air from the path of the projectile; overcoming the resistance to skin friction between the surface of the projectile and surrounding particles of air; formation of eddies around the projectile; formation of a partial vacuum in rear of the projectile; setting up and overcoming a wave motion in the air; and gyroscopic wobbling.

■ 14. BALLISTIC COEFFICIENT *C*.—The retardation formula in use at the present time for computing trajectories contains, among other factors, one called the "ballistic coefficient," represented by the letter "*C*". This term represents a measure of the ability of the projectile to overcome air resistance and maintain its velocity. The ballistic coefficient is usually expressed as—

$$C=\frac{w}{id^2}$$

where w is the weight of the projectile in pounds, d is the diameter in inches, and i is a coefficient dependent upon the shape of the projectile, location of rotating band, and observed characteristics of flight of the projectile. It has been found that these properties have considerable effect on the retardation.

■ 15. TRAJECTORY IN AIR.—The trajectory in vacuo is dependent only on the initial velocity and quadrant angle of departure (par. 12c). The equations of the trajectory in air contain not only these factors but also factors based on the value of the ballistic coefficient, the rotation imparted to the projectile, and existing atmospheric conditions. As a result, there is considerable change in the characteristics of the trajectory as may be seen from the following summary:

a. The trajectory is no longer a parabola.

b. The trajectory is no longer symmetrical; the descending branch is shorter, more curved, and takes longer to traverse than the ascending branch, and the quadrant angle of fall is larger than the quadrant angle of departure.

c. The trajectory no longer depends on the initial velocity and the quadrant angle of departure only; its shape is affected by the weight and shape of the projectile.

d. Terminal velocity is less than initial velocity.

e. Maximum range is not necessarily attained at a quadrant angle of departure of 45°.

f. The trajectory does not lie in the plane of departure. This is due to air resistance and rotation of the projectile and is called "drift." (See par. 38.)

CHAPTER 3

FIRING TABLES

SECTION I

GENERAL

■ 16. OBJECT.—*a*. The object of firing tables is to present in convenient form the data necessary to the artilleryman in computing firing data for his guns. The Ordnance Department computes and publishes these tables for each combination of gun and ammunition used in the service.

b. In order to prepare firing tables, trajectories are computed for various quadrant elevations of a gun, and firings are conducted at the proving grounds with the gun at these elevations. Computed trajectories and trajectories actually obtained are compared and computations are adjusted and tabulated, data for other elevations being completed by interpolation. This tabulation sets forth the range-elevation relation for the gun and ammunition used in the firing and is the most exact of any data included in the tables. Certain of the data desired cannot be obtained from measurements and consequently must be computed. In general, the principal elements now determined by measurements in proving ground firings are the initial (or muzzle) velocity, quadrant angle of elevation, quadrant angle of departure, jump, range attained, and drift. The computed elements are the maximum ordinate, time of flight, angle of fall, and terminal velocity.

■ 17. CONTENTS.—The present standard firing tables are published in book form. The introduction contains a table of contents; general information about the gun, carriage, and ammunition; an explanation of the tables; an explanation of the meteorological message; and an example of the use of the firing tables in computing firing data. This introduction will be of material benefit when using the tables and should be consulted freely. The firing tables follow the introduction and are divided into two parts. Part 1 contains charts and tables giving information of a general character, such as determination of range and deflection components of the ballistic wind. Part 2 contains the data applicable to a particular combination of cannon, powder charge, projectile, and fuze. The range-elevation relation and elements of the trajectory already referred to are listed first in table A, followed by several tables of differential effects which are included for the purpose explained in paragraph 18. (Table A of some firing tables, 155–B–4, for example, contains some differential effects.) Frequently, additional parts are included to cover additional combinations of cannon and ammunition.

■ 18. STANDARD BALLISTIC CONDITIONS.—In order to compare the results of firings at different times and places and take into account conditions that actually exist at the time and place of firing, range-elevation relations are constructed for certain assumed ballistic conditions called standard. Observations may then be taken at the time and place of the firing and, by the use of the tables of differential effects, corrections may be made to adapt the firing data to the nonstandard conditions measured. The most important of the standard ballistic conditions are based on the following assumptions:

a. The earth is motionless.

b. The gun and target are at the same altitude above sea level.

c. Muzzle velocity for which the firing tables are constructed (that is, standard muzzle velocity) is actually developed.

d. Powder temperature is 70° F.

e. Weight of the projectile is as listed.

f. There is no wind.

g. Atmospheric temperature is 59° F. at the muzzle and varies regularly with the altitude in a particular manner.

h. Atmospheric density varies regularly with the altitude according to certain fixed laws and is equal, at the gun, to that density obtaining when the temperature is 59° F., barometric pressure 29.528 inches, and the air 78 percent saturated with moisture.

i. Drift (including lateral jump) is as determined by experimental firing.

j. Vertical jump is as determined by experimental firing.

k. Ballistic coefficient is a constant for any particular trajectory and is as determined by experimental firing.

l. Action of gravity is uniform in intensity, is directed toward the earth's center, and is independent of the geographical location of the gun. The acceleration due to gravity is 32.152 feet per second per second.

m. Certain assumptions are made as to the retardation of the projectile by the atmosphere, which include those that the retardation is proportional to the air density, to the reciprocal of the ballistic coefficient, and to a tabulated function of the velocity.

■ 19. NECESSITY FOR CORRECTIONS DUE TO NONSTANDARD CONDITIONS.—Conditions at the gun position at the time of firing can never be exactly the same as those considered as standard. Their variations from standard must therefore be determined and corrected for. The following sections will be devoted to a brief discussion of such corrections and the manner of making them. All of the assumptions which are known to be erroneous and for which corrections are necessary will be discussed and, in addition, mention will be made of several other factors which influence the actual trajectories obtained. Assumptions in paragraph 18*a* to *h* are not usually true and corrections for them are necessary. The manner of taking assumptions *i* and *j* into account will be described. No particular discussion of the three remaining assumptions, *k*, *l*, and *m*, will be made as these do not enter into the calculation of firing data. However, indirect reference is made to assumptions *k* and *m* in connection with atmospheric conditions discussed in section VII.

SECTION II

CORRECTIONS DUE TO ROTATION OF THE EARTH

■ 20. EFFECTS OF ROTATION.—Rotation of the earth affects location of the point of impact in both range and direction.

A mathematical explanation of these effects is reasonably simple. Physical explanations, however, become difficult because two complex motions must be considered simultaneously; that of a chord of a great circle of a sphere (the X-axis of the reference system) rotating with the surface of that sphere at a constant speed, and that of a body (the projectile) moving in the path of an ellipse with a nonuniform motion. No rigid physical explanation will be undertaken. However, some understanding of the nature of the important causes and effects may be obtained from the concepts given below. Air resistance will be neglected in this discussion on the ground that it will cause little alteration in the result.

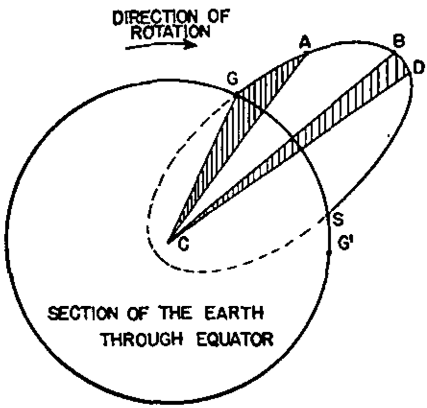


FIGURE 3.—Motion of a satellite.

a. Motion of a satellite.—The projectile in its flight becomes a satellite of the earth and is independent of any of its motion except that of its center of gravity. Assume the trajectory in vacuo and the extreme case of a gun fired vertically upward at the equator. The earth rotating toward the east imparts to the projectile an eastward velocity in addition to the upward velocity imparted by the gun. The projectile in assuming the motion of a satellite describes a portion of an ellipse in vacuo which, if continued through the earth, would follow some such path as shown to an exaggerated degree in figure 3.

Kepler's Second Law of orbital motion as applied in this case provides that a line joining the center of the earth and the projectile must sweep out equal areas in equal times. If the areas *GCA* and *BCD* are equal, the paths *GA* and *BD* are traversed in equal times. Therefore, the radius vector *BC* must be moving more slowly than it was at the point *G*. When the projectile reaches the point *S*, the radius vector is again moving at the same rate as at *G*, that is the velocity of a radius of the earth. At all intermediate points on the trajectory, the radius vector has been moving more slowly than the earth's radius and as a consequence the gun will have moved to some point *G'* during the time of flight. This effect is a result of the eastward rotation of the earth; it exists at all angles of elevation and increases with the angle of elevation; it exists at all azimuths of fire; and its sense is always westward.

b. Rotation of reference system.—It was shown in *a* above that the trajectory is independent of any of the earth's motion except that of its center of gravity. Consequently, the position of the trajectory in space is not affected by the earth's rotation. The reference system (rectangular axes) upon which the trajectory was based and calculated is affected by such rotation; it rotates to the eastward with the earth. This has the effect of causing the actual level point to rotate to the eastward of the computed (or expected) level point. As an illustration, if from a position at sea level, a projectile is fired eastward at the azimuth of a star at the instant the star appears on the horizon, then at the end of the time of flight, computed for a motionless earth, the projectile instead of reaching the ground will have an azimuth and angle of site equal respectively to those of the star. From the point of view of an observer at the gun, the projectile's trajectory will have been raised and consequently the range is increased. One might visualize this effect as altering the curvature of the earth; for a projectile fired to the east, it increases the effective curvature and the range attained and for a projectile fired to the west, it decreases the curvature and the range. This effect is always present when there exists an eastward (or westward) component of muzzle velocity (in addition to the eastward velocity imparted by the earth's rotation).

It is always to the eastward and exceeds the satellite effect at angles of elevation less than about 60°.

c. Spherical shape of the earth.—Since the earth is spherical, the linear eastward velocity due to rotation is greatest at the equator and decreases as the latitude increases until at either pole it becomes zero. A projectile has the same eastward velocity due to the earth's rotation as the point from which it left the earth. If it is fired toward a point having less eastward velocity, for example, from a point in the northern hemisphere toward the north pole, it will have a greater eastward velocity than the expected point of fall. The actual point of fall will therefore be to the eastward of the expected point of fall. On the other hand, if the projectile is fired toward a point having more eastward velocity due to rotation, for example, from a point in the northern hemisphere toward the equator, the actual point of fall will be to the westward of the expected point of fall. This effect is always present when there exists any northward (or southward) component of muzzle velocity; it varies in amount with the latitude of the piece. It may be either eastward or westward, depending on the latitude of the gun and the direction of fire.

d. The resultant of these three principal effects is either to the eastward or the westward, depending upon the amount of each. Its value and sense depend upon the direction of fire, latitude of the gun, and characteristics of the trajectory. It may influence either the range or the direction or both, depending on the direction of fire. Tables of differential effects (tables *E* and *K*, part 2, of firing tables), from which the effects may be found, are provided in the firing tables of large guns and howitzers. These tables are omitted from the firing tables for short-range cannon on which the effects are negligible.

■ 21. APPLICATION.—Provisions are made on the range correction board M1 and the deflection board M1 for the application of rotation corrections when appreciable. In the absence of such equipment, the firing tables may be used. The tables are entered with latitude of gun position, range to target, and azimuth of target as arguments, and the corresponding effects determined.

■ 22. EXAMPLES.—Assume a 12-inch seacoast gun, M1895, on barbette carriage, M1917, at latitude 30° south, firing a 975-pound projectile at a target at 25,000 yards range and 50° azimuth from south.

a. What is the range effect of rotation of the earth?

Solution: In table *E*, part 2, Firing Tables 12–F–3, we find that the azimuths given are from north, so the azimuth of the target must be referred to north, giving 230°. From the section for 30° of latitude the data tabulated below are extracted and the range effect is found by double interpolation:

Latitude 30° (north or south)

Ranges	Azimuths—degrees		
	225	230	240
24,000	−71	−76	−87
25,000		−77	
26,000	−72	−77	−88

The range effect is therefore −77 yards.

b. What is the lateral effect of rotation of the earth?

Solution: Using the same converted azimuth and other arguments, table *K* is consulted and the following data are extracted for interpolation:

Latitude 30° south

Ranges	Azimuths—degrees		
	210	230	240
24,000	+1.2	+1.3	+1.4
25,000		+1.4	
26,000	+1.3	+1.4	+1.5

NOTE.—The sign of the effect obtained from the firing tables is *plus* which signifies that the effect is to the *left*.

Deflection effects should be taken to the nearest 0.01°. Therefore the effect in this case is *left* 1.4 mils or *left* 0.08°.

SECTION III

CORRECTIONS DUE TO HEIGHT OF SITE

■ 23. EFFECT OF DIFFERENCE IN ALTITUDE.—*a.* The term "height of site" is used to represent the altitude of a gun above the assumed datum level (sea level at mean low water). Corrections for height of site are really corrections for the difference in altitude between the gun and the target. They are made necessary because of assumption *b*, paragraph 18, that the gun and target are at the same altitude, which means that for a given quadrant elevation the range listed opposite that elevation in the firing tables is the range *GB* (fig. 4) measured along the surface of a sphere concentric with the earth. (See note below.) The point *B* is called the "level

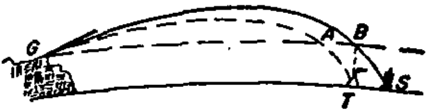


FIGURE 4.—Effect of target below gun.

point." Thus, if a gun is above the surface of the sea, the measured ranges to all targets on the sea must be transformed into level point ranges before the elevations necessary to hit the target can be determined. The corrections necessary to make the transformation are for range only; no deflection corrections are involved. When the target is above the level of the gun, the effect of the difference in altitude is to cause the projectile to fall short, and when the target is below the gun, the effect is to cause the projectile to fall over.

NOTE.—According to this assumption, the range to the target should be measured as a curved range. The ranges measured by the standard position-finding systems are never curved ranges. but the error made in assuming they are curved ranges is negligible for all distances involved in artillery firing. Therefore range corrections for curvature of the earth are never necessary, and as an argument in entering the firing tables a range obtained from the plotting board may be used.

b. In figure 4, *T* is the target on the surface of the sea, *GB* is the range to the target, and *B* is the level point for the trajectory *GBS* that corresponds to that range in the firing table. The effect of the difference in altitude (*TB* approximately) is the distance *TS*. *A* is the level point for the trajectory *GAT* that passes through the target. The range for this trajectory is *GA*. Therefore the distance *AB* is the value that must be subtracted from the range *GB* to correct for target below gun, and is the value listed in the tables of differential effects. The effect of a target above the gun may be explained in a similar manner. Separate tables are included in part 2 of most firing tables for target below gun (table *B*) and target above gun (table *C*). The correction for a given situation may be found by entering the proper table with range and height of target as arguments. Since the values are tabulated as effects, the signs must be changed before applying them to the range. The table of differential effects mentioned is not contained in Firing Tables 155–B–4. Therefore, when using this set of tables, it is necessary to calculate a correction to angle of site as illustrated in the introduction of that publication.

■ 24. Determination of Difference in Altitude.—The difference in altitude that is used in making the corrections is the distance between the spherical surfaces containing the gun and the target. If an accurate map is available, it may be taken from the contours of the maps. If it is necessary to measure the angle of site and compute the difference in altitude, then a correction should be made for the effect of curvature of the earth and refraction on the line of sight. (See par. 58.) Their combined effect is to cause the point sighted on to appear to be above its true position by an amount approximately equal to h where

$$h \text{ (in feet)} = 0.18 \times (\text{thousands of yards range})^2$$

This correction is always additive if applied to the computed altitude of the new point. The sign is not constant if the correction is applied to the difference in altitude.

Example: A battery of 155-mm guns is to fire at a target whose range, *R*, from the guns, as measured on the plotting board, is 7,400 yards. The vertical angle (ϵ) to the target, measured from the horizontal is $-0°8'0''$. What is the difference in altitude between the guns and the target?

Solution:

Apparent difference in altitude (in feet) $=3\ R$ (in yards) $\times \tan \epsilon$

$$\begin{aligned} \log 3 &= 0.47712 \\ \log 7{,}400 &= 3.86223 \\ \log \tan \epsilon &= 7.36682-10 \end{aligned}$$

log apparent difference in altitude $=1.70617$

Apparent difference in altitude $=51$ feet.

The actual position of the target is below its apparent position by an amount h, the combined effect of curvature and refraction.

$$h=0.18\times(7.4)^2=10 \text{ feet.}$$

Therefore the target is $51+10$ or 61 feet below the guns.

■ 25. TIDE.—The datum level from which altitudes are measured is usually sea level at mean low water. If the target is on the surface of the sea, the altitude of the gun above the target is affected by the tide, and a correction for it must be made. It may be included in the height of site correction.

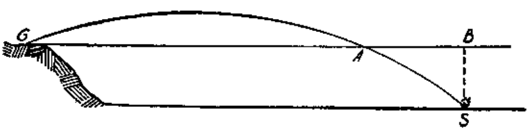


FIGURE 5.—Expected range from given elevation.

■ 26. EXAMPLES.—*a*. A 12-inch gun (**FT** 12–F–3), firing 975-pound A. P. projectile, is to be fired from a position 200 feet above target at a map range of 15,200 yards. What corrected range should be used assuming that all other conditions are normal?

Solution: Entering table B, part 2 of the firing tables, the range effect for a target 200 feet below gun is found to be +245 yards. Therefore corrected range is 15,200—245 yards or 14,955 yards.

b. The converse of this problem may be solved. Assume that the 12-inch gun described above is to be fired at the target and an elevation corresponding to a level-point range of 14,955 yards is used. Assume all conditions to be normal except that the gun is 200 feet above the target. What will be the expected range to the splash?

Solution: From the assumptions, the level-point range *GA* in figure 5 is 14,955 yards. Enter table B, part 2 of the firing tables, with 200 feet as an argument. We know that the range *GB* to the splash will be greater than 14,955 yards. We also know that the range *GB* minus the correction *AB* must equal 14,955 yards. An inspection of table B shows that it lies somewhere between 15,000 and 15,500 yards as follows:

Map ranges	15,000	R	15,500
Range correction	—250	$-\Delta R$	—237
Level point range	14,750	14,955	15,263

R will be the same proportional distance between the two map ranges as 14,955 is between the two level-point ranges, that is

$$\frac{R-15{,}000}{15{,}500-15{,}000}=\frac{14{,}955-14{,}750}{15{,}263-14{,}750}$$

$$R=15{,}000+\frac{205}{513}\times 500=15{,}200 \text{ yards.}$$

c. Assume that a 155-mm gun, firing shell, HE, Mk. III, with fuze, short (Mk. IV*), is emplaced 406 feet above datum level, and is to fire at a destroyer at a range of 6,000 yards from the gun. The tide is +6 feet. Assume that all other conditions, except the difference in altitude, are standard and that the corrected range for use with the normal powder charge is to be found. What is the corrected range?

Solution: The difference in altitude is 400 feet or 133 yards. The range is 6,000. 133/6,000=22.2 mils. The target is below gun; therefore the site is —22.2 mils. To correct the site for nonrigidity of trajectory, enter Firing Tables 155–B–4, table A, part 2b–1, opposite range 6,000. The correction for —1 mil angle of site is —0.02 mils. 22.2×—0.02 mils=—0.4 mils. The corrected site is then —22.6 mils. The elevation for a level-point range of 6,000 yards is 123.6 mils. Corrected elevation is 101.0 mils. This corresponds to a corrected range of 5,230 yards.

■ 27. APPLICATION.—Corrections for height of site are applied on the range correction board when such an instrument is used. For mobile seacoast artillery, the corrections for both height of site and tide are made on the board. Fixed seacoast artillery weapons, except 12-inch mortars and guns provided with M5 data transmission sets, are equipped with range disks whose graduations are corrected for the known height of site of the battery above the datum level. On these guns, only corrections for tide need be made on the range correction board. Fixed guns equipped with electric data transmission sets do not have range disks, and corrections for both height of site and tide are made on the board. Mortars have such a large angle of fall that the height-of-site correction is not appreciable and is therefore neglected.

SECTION IV

CORRECTIONS DUE TO EXPECTED VARIATIONS IN MUZZLE VELOCITY

■ 28. POWDER TAG VELOCITY.—The corrections made necessary by assumptions *c* and *d*, paragraph 18, are discussed in this section. The muzzle velocity is one of the factors that influence the shape of the trajectory and therefore the range. A definite value of this velocity must be assumed before the trajectory and the firing tables can be computed. It is called the "standard muzzle velocity" and is listed plainly in all firing tables. Then, if the velocity that the powder is expected to develop can be determined, corrections can be made for the variation from standard. Consequently, each lot of powder is proof-fired by the Ordnance Department before being issued to the service, its velocity is measured by chronograph, and the charge is altered if necessary to bring it to the standard velocity. A powder tag with the velocity at standard temperature (70° F.) and the lot number listed on it is tied to each charge.

■ 29. TEMPERATURE OF POWDER.—*a*. The temperature of powder affects the rate of burning of the charge. For a given powder charge, the higher the temperature the higher is the expected velocity. Since the firing tables are constructed on the assumption that the powder temperature is a particular value, that is, 70° F., it is necessary to determine the temperature at the time of the firing and correct for the variation from standard. In the concrete magazines of fixed armament, the temperature of the magazines does not vary greatly from hour to hour and can be taken as the temperature of the powder stored therein if it has been there for 2 weeks or more. In the field, the temperature can be obtained from a thermometer inserted in a powder container if it has been there over an hour. It is sufficient to take the temperature of one charge as that of a group of charges stored together under like conditions.

b. The effect of variations of temperature on the muzzle velocity may be obtained from a chart included in the firing tables. The chart or table should be entered with temperature to the nearest degree and the percentage change should be taken to the nearest 0.1 percent or nearest foot-second.

Example: Given a battery of 12-inch guns, M1895, on barbette carriage, M1917, using a 975-pound projectile (FT 12–F–3). Assume that the temperature of the powder is 85° F. and the powder tag velocity is 2,200 f/s. What is the corrected powder tag velocity?

Solution: From the temperature-velocity chart, the percentage change for 85° F. is +1.6 percent. The corrected velocity is 2,200+35=2,235 f/s.

c. When powder lots are proof-fired, the powder tag velocities are transformed to those at standard temperature by the use of the same chart.

Example: Assume that the temperature of the powder at a proof-firing of one of the guns of the preceding example is 80° F. and the developed muzzle velocity is 2,222 f/s. What is the velocity at standard temperature?

Solution: From the chart, the percentage change is 1 percent. Therefore the velocity at standard temperature is 2,222/101=2,200 f/s.

■ 30. ASSUMED VELOCITY FOR USE ON RANGE CORRECTION BOARD.—Before a firing can be started, a muzzle velocity for use on the range correction board must be assumed. In the absence of other data, the muzzle velocity given on the powder tag should be used. However, the muzzle velocity developed by the guns of a battery may be different from that indicated on the powder tag. Therefore, if the developed velocity of the powder when fired in the same guns is at hand and there is no reason to question the reliability of this determination, it should be used in preference to the powder tag velocity. Reports of previous firings on W. D., C. A. C. Form No. 25 (Matériel and Powder Report) show for each gun its developed muzzle velocity at standard temperature. A base piece should be selected and the developed muzzle velocity of that gun used as the assumed velocity. The assumed velocity should then be transformed to that corresponding to the temperature of the powder and used in the selection of the muzzle velocity curve on the range correction board.

■ 31. EXAMPLES.—*a*. Assume that preparations are being made to fire a 12-inch gun, M1895, on barbette carriage, M1917, using a 975-pound projectile (FT 12–F–3), that the powder lot selected for the firing has a powder tag velocity of 2,275 f/s, that the same powder lot was used in a shoot held 2 years previously and developed a velocity of 2,260 f/s at standard temperature, and that the powder temperature is 80° F. Which muzzle velocity curve on the range-correction board should be used?

Solution: The muzzle velocity curve that should be used is that corresponding to an assumed muzzle velocity of 2,260 f/s corrected for a temperature of 80° F. or 2,260+0.01 (2,260) = 2,283 f/s.

b. Assume that fire is properly prepared and that after completion of firing, the observed results indicate a center of impact 260 yards short of the target. The mean range to the target is 19,500 yards. What is this deviation in terms of muzzle velocity? What was the developed muzzle velocity at 80° F.?

Solution: From table Fb, part 2, of the firing tables, this deviation is equivalent to a decrease in muzzle velocity of 20 f/s.

	10 f/s	19. S	20 f/s
19,000	128		257
19,500	131	260	263
20,000	134		268

The developed velocity at 80° is 2,283—20=2,263 f/s.

c. What is the developed muzzle velocity at standard temperature (70° F.)?

Solution: The developed muzzle velocity at standard temperature is 2,260—20 f/s or 2,240 f/s.

NOTE.—Since the field method of determining the developed muzzle velocity is only approximate, it is not necessary to apply the 20 f/s variation to the assumed velocity at actual temperature and reduce it to standard temperature; the variation may be applied directly to the assumed velocity at standard temperature as shown.

SECTION V

CORRECTIONS DUE TO VARIATIONS IN WEIGHT OF PROJECTILE

■ 32. GENERAL.—*a.* Variations in the weight of the projectile have two effects which are contradictory. An increase in the weight will tend to cause a decrease in the range due to a decrease in muzzle velocity and at the same time it will tend to cause an increase in the range due to increase in the ballistic coefficient (par. 14). The net effect is to decrease the range at shorter ranges and to increase the range at longer ranges. The value of the net effect and the point where the effect changes sign depend on the gun, projectile, and angle of elevation. In some cases, the range at which the effect would change sign is beyond the maximum range of the matériel. A decrease in the weight of the projectile has opposite effects.

b. Firing tables contain the effects due to variations in the weight of the projectile. The effect in yards of range may be found by entering the tables with the range and weight (or the variation in weight) as arguments. For example, assume that the average weight of the projectiles for a particular firing with 12-inch guns (standard weight 975 pounds) is 965 pounds and that the range is 16,000 yards. From table D, part 2, Firing Tables 12–F–3, the effect is +21 yards. When a range correction board is used, the correction is made by means of curves on that instrument.

SECTION VI

CORRECTIONS DUE TO WIND

■ 33. GENERAL.—Assumption *f*, paragraph 18, states that there is no wind. This is true only in exceptional cases. With the exception of a wind blowing along the line of fire or perpendicular thereto, all winds have two effects on the projectile; an effect on the range and an effect on deflection. These effects have been evaluated for the different types of projectile and are listed in part 2 of the firing tables. The data on the ballistic wind, used in entering the tables, are contained in the meteorological message which is available for all firing. Having ascertained the azimuth and velocity of the ballistic wind, it may be resolved into its two components, range and deflection, on the wind component indicator. Those components may then be applied on the range correction board and the deflection board and wind corrections made by the normal operation of those boards. For checking the accuracy of such instruments, means are provided in the firing tables for making the computations. Part 1 contains either a wind component chart or a table from which the two components may be found. The chart direction of the wind must first be determined. This is done by subtracting the azimuth of the plane of fire from the azimuth of the ballistic wind, *both expressed in mils from zero north*. The azimuth of the wind may be increased by 6,400 mils if necessary. The wind component chart provides a graphical means of transforming the polar coordinates of *chart direction* (vectorial angle) and *wind velocity* (radius vector) into rectangular coordinates of *range component* (ordinates) and *deflection component* (abscissas). The wind component table provides a tabular means of doing the same thing but gives the components for a 1-mile wind only. This may be converted to the proper value by multiplying by the velocity taken from the message. The wind effects may then be found by entering firing tables with the range and the proper wind component as arguments. Since each table is used for both plus and minus winds, care must be used in choosing the sign of the effects.

■ 34. EXAMPLES.—*a*. Given: azimuth of plane of fire=90° from south; azimuth of ballistic wind=800 mils from north; velocity of ballistic wind=6 m. p. h. What is the chart direction of the wind?

Solution:

		Mils
Azimuth of wind	=800	
Add	6, 400	
		7, 200
Azimuth of plane of fire	=90°	
	Mils	
	=1, 600	
Add	3, 200	
		4, 800
Chart direction of wind		=2, 400

b. Assume that a battery of 155-mm guns is to fire at a target at a range of 12,000 yards, using normal charge and shell HE, Mk.III, fuze, short, Mk.IV*. The chart direction of the wind is 2,400 mils and the velocity is 6 m. p. h. What are the range and deflection effects of the wind?

Solution: From the wind component table, part 1, Firing Tables 155–B–4, the range wind component is +4 m. p. h. (rear wind) and the cross wind component is (left) +4 m. p. h. (to the nearest mile per hour). From column 18, part 2b–1, the range effect is +53 yards and from column 12. the deflection effect is +2 or left 2 mils.

NOTE.—Direction of the effect should be deduced from a sketch of the situation. Signs of the deflection effects in the firing tables are for the sights of field artillery matériel, while sights on coast artillery guns are graduated in the opposite manner. Therefore, to avoid confusion and errors, the direction of the effect should always be determined as *left* or *right* and not minus or plus.

c. Assume that a battery of 12-inch guns (FT 12–F–3) is to fire at a range of 16,000 yards when a 10-mile wind is blowing from a chart direction of 4,000 mils. What are the range and deflection effects of the wind?

Solution: From the wind component chart, part 1 of the firing tables, the range component is +7 m. p. h. (rear wind) and the cross wind component is right 7 m. p. h. From table I, part 2, the range effect is +33 yards, and from table J, the deflection effect is 0.07° or right 0.07°.

SECTION VII

CORRECTIONS DUE TO NONSTANDARD ATMOSPHERIC CONDITIONS

■ 35. GENERAL.—Assumptions *g* and *h*, paragraph 18, relate to a standard atmosphere. The condition of the atmosphere is seldom standard and corrections must be made for any variation from standard. Of the various meteorological characteristics of the atmosphere exclusive of actual rainfall, only two are regarded as significant for purposes of artillery fire. They are the density and the temperature of the air. Corrections are made for the effect of these two variable conditions on the range. The moisture present as vapor affects the density of the air and therefore the correction applied. However, the effects on the flight of the projectile due directly to variations in the moisture content of the air are not at present corrected for but are still being investigated.

■ 36. DENSITY.—*a*. The density of the air measures the mass that must be displaced by the projectile. The greater this mass, that is, the greater the density, the more will be the energy absorbed in overcoming it and consequently the less will be the range attained. The result is that for a density greater than normal the range will be decreased and for a density less than normal the range will be increased. The density also varies with the altitude above the ground, decreasing as the altitude increases. This factor is especially important and varies with the maximum ordinate of the trajectory.

b. The meteorological message gives for the maximum ordinate in question the ballistic density in percent of normal. The ballistic density is a fictitious constant density which would have the same total effect on the projectile during its flight as the varying densities actually encountered. This ballistic density is calculated with reference to the altitude of the meteorological datum plane and must be corrected for the difference in altitude between the datum plane and the battery. This can be done by means of the density formula appearing in part 1 of the firing tables, which states that for an increase of 100 feet in altitude the density decreases 0.3 percent and vice versa. The ballistic density in percent of normal so corrected is then applied to the range correction board which mechanically determines the proper range correction. To check the accuracy of the correction thus applied, part 2 of the firing tables may be entered with the ballistic density expressed as a percentage increase or decrease from normal to find the resulting effect on the range.

■ 37. TEMPERATURE.—*a*. The effect on range due to temperature is called the elasticity effect. The temperature of the air has an effect on the air resistance by influencing the elasticity, which in turn influences the velocity of the wave motion set up by the projectile. (This wave motion is possible because of the elasticity of the air; its velocity is that of sound.) The effect of the wave motion on the projectile is dependent on the relation of the velocity of the projectile to the velocity of the wave motion. As the velocity of the wave motion is influenced by the air temperature, the air resistance is influenced and consequently the range.

b. With some guns the remaining velocity of the projectile never gets as low as the velocity of the wave motion, while with others it never gets as high as the velocity of the wave motion. With the former, the range effect for a decrease of temperature is usually positive, and for an increase, negative; with the latter the converse is the case.

c. With some guns, the remaining velocity of the projectile passes through the velocity of the wave motion. In this case, the net range effect for a decrease in temperature may be either positive or negative, depending on the time that the velocity of the projectile was greater than the velocity of the wave motion and the time that it was less. For a particular gun, these times will depend on the shape of the trajectory; that is, on the elevation or range. Therefore, for some ranges (a particular gun being considered) the range effect for a decrease of temperature is positive and, for other ranges, negative; the converse is the case for an increase of temperature. The point where this change of sign occurs depends on the matériel. For some matériel, the ranges do not extend to the point where a change of sign occurs.

d. When the temperature is not standard (59° F.), an elasticity correction is necessary. The temperature at the battery may be observed by a thermometer or it may be taken from the meteorological message. In the latter case, it must be corrected, if there is a difference in altitude between the meteorological datum plane and the battery, by using the thermometric formula in part 1 of the firing tables. The formula states that for every 100 feet increase in altitude the temperature decreases ⅕° F., and vice versa. The temperature at the battery is applied on the range correction board; the mechanical correction thus obtained may be checked by entering part 2 of the firing tables with arguments of range and temperature to find the corresponding effect on the range.

SECTION VIII

CORRECTION DUE TO DRIFT

■ 38. GENERAL.—To obtain stability in flight, an elongated projectile fired from a modern gun is given a motion of rotation about its longer axis by means of the rifling of the bore. The resistance of the atmosphere to the movement of such a projectile so rotating causes it to deviate from its original plane of direction. This deviation is called "drift."

■ 39. CAUSE OF DRIFT.—*a*. The principal cause of drift is gyroscopic action. (A minor component of drift is caused by air viscosity acting in the same manner by which it curves a rotating baseball.) By gyroscopic action a projectile tends to maintain a constant axial direction (line of departure). Since the trajectory curves, the axis of the projectile thus fails to follow the tangent to the trajectory and a center of air pressure is built up on the underside of the point (near the bourrelet) and in advance of the center of gravity. A turning movement is thus set up tending to lift the point of the projectile. Such a turning movement when exerted on the axis of a gyroscope produces a motion about the point of support, *not in the direction of the force but at right angles to it*, and in a direction dependent upon the direction of rotation of the gyroscope. This gyroscopic effect causes the axis of the projectile to veer toward the right about the center of gravity (center of support) when the twist is to the right, causing a rudder action which in turn curves the trajectory to the right.

b. Drift with the guns in our service is, with one exception (the 37-mm subcaliber gun), to the right, the rifling having a right-hand twist. (The 37-mm subcaliber gun has a left-hand twist and a drift to the left.) Drift varies with the projectile, muzzle velocity, elevation, and speed of rotation of the projectile. The amount of drift is determined at the proving ground by experimental firing and, combined with lateral jump, is tabulated in firing tables in angular units as lateral effects of drift.

■ 40. APPLICATION.—Provision is made on the deflection board for applying the drift correction to the firing data. If such an instrument is not available, the firing tables may be used. In some firing tables, drift effects may be found in table A, part 2; in others they are listed in table J.

SECTION IX

OTHER CONDITIONS AFFECTING FLIGHT OF PROJECTILE

■ 41. JUMP.—When a gun is fired, it does not remain steady but jumps through a small angle both vertically and laterally. Thus the line of departure does not coincide with the axis of the bore when laid. Jump is due to numerous factors, not all of which are clearly understood, and is not constant. Among other things, the general design and stability of the gun and carriage, as well as the elevation, influence the jump. Jump is measured at the proving ground for several elevations and a jump curve plotted so that its value for all other elevations can be obtained. It is of no particular importance insofar as the calculation of firing data is concerned, because no account of it need be taken in this calculation. The vertical jump is included in the elevation and the lateral jump in the drift as tabulated in the firing tables.

■ 42. MISCELLANEOUS EFFECTS.—*a*. Assumptions *k* and *l*, paragraph 18, pertaining to the ballistic coefficient and to the action of gravity, are of no interest for practical purposes, their effects having been taken into account as fully as possible in the construction of the firing tables. No means are provided for correcting for these effects individually.

b. There are other miscellaneous effects not due to the firing table assumptions which may enter into the problem of preparation of firing data; for example, the effect caused by the displacement of gun sights from the pintle center of the guns and that caused by lack of level of the trunnions (called cant). However, since they do not pertain to the construction and use of the firing tables, they will not be discussed in this section. Those effects which are of practical importance are discussed in later chapters.

CHAPTER 4

CALCULATION OF FIRING DATA

■ 43. GENERAL.—*a.* The calculation of firing data for seacoast artillery usually involves the use of various mechanical devices in the plotting room. In order to understand the operations performed by these instruments and to be able to check the results obtained from them, the artilleryman must be entirely familiar with the mechanics of calculating firing data by means of the firing tables only.

b. After the uncorrected range and azimuth have been determined, the preparation of firing data is completed by applying corrections to these data for all known nonstandard conditions.

■ 44. CLASSIFICATION OF CORRECTIONS.—Nonstandard conditions and corrections therefor have been discussed in chapter 3. They are also discussed in detail in the general information published as an introduction to each set of firing tables. For convenience, a tabulation is made here. A convenient classification is as follows:

a. Range corrections.

(1) *Position corrections.*

Rotation of the earth (when necessary).
Height of site and tide.
Gun difference.[1]

(2) *Matériel corrections.*

Weight of projectile.
Muzzle velocity.
Calibration correction.[1]
Trunnions out of level or quadrant in error.[1]

(3) *Weather corrections.*

Temperature of powder.
Wind.
Density of the air.
Temperature of the air (elasticity effect).

[1] These corrections are for individual guns. Other corrections apply to all the guns of a battery.

b. Lateral corrections.

(1) *Position corrections.*
Rotation of the earth (when necessary).
Parallax.[1]

(2) *Matériel corrections.*
Drift.
Trunnions out of level.[1]

(3) *Weather corrections.*
Wind.

■ 45. Accuracy of Computations.—By using the firing tables and tables of logarithms, it would be possible to determine the different ranges, azimuths, and effects of nonstandard conditions to a great degree of refinement. But it would obviously be absurd to determine the ranges to tenths of yards when the coordinates from which they have been determined may be in error by whole yards; or to correct for a fraction of a foot per second of muzzle velocity when the original determination may have been in error by 1 to 5 feet per second. There is rarely any justification for exceeding the following limits of accuracy in the deliberate computation of firing data:

	Solve to or use nearest—
Firing range	10 yards.
Firing elevation	1 mil or minute.
Firing azimuth or deflection	0.01°.
Range for determining differential effects	100 yards.
Range effects, distances in all calculations	1 yard.
Lateral effects	0.01°.
Latitude of gun	1°.
Azimuth of target (rotation or wind)	1°.
Height of site	1 foot.
Weight of projectile	1 percent.
Muzzle velocity	1 foot/second.
Component of wind	1 mile per hour.
Density	1 percent.
Temperature (air or powder)	1° F. or C.

Use 5-place logarithm tables for preliminary calculations.

[1] These corrections are for individual guns. Other corrections apply to all the guns of a battery.

■ 46. EXAMPLES.—*a. Case III pointing.*—Given a battery of 12-inch guns, M1895, on barbette carriage, M1917, firing 975-pound A. P. projectile with a tabular muzzle velocity of 2,275 f/s (FT 12–F–3).

Directing point: No. 1 gun.

Azimuth and distance from No. 1 to No. 2 gun: 81.45° (zero south), 106 yards.

Clinometer tests show that the base ring of No. 2 has settled so that near azimuth 200°, 1 minute must be added to the elevation.

Latitude: 42° north.

Altitude of battery: 430 feet above mean low water.

Height of tide: +10 feet.

The calibration records indicate that No. 2 develops 5 f/s more than No. 1.

The indices are set so that the guns are laid parallel for the same azimuth setting.

Weight of projectiles: 965 pounds.

Powder tag velocity: 2,275 f/s.

Muzzle velocity developed by No. 1 gun in previous firings: 2,257 f/s.

Temperature of powder: 62° F.

Meteorological message:

M S L M S L
30262
0620799
1620898
2600997
3601195
4590992

Azimuth of set-forward point: 200.60°.

Range of set-forward point: 15,840 yards.

Required: Firing ranges and azimuths for checking mechanical solution of range section.

Solution: See computations on form below:

FORM FOR CALCULATION OF FIRING DATA, SEACOAST ARTILLERY

12 —inch Gun, M1895, Std. MV 2275 f.s., Std. Wt. Proj. 975 lbs., Firing Tables 12-F-3

PRELIMINARY CALCULATIONS

GUN-TARGET

Target X' =		Target Y' =	
Gun X_g =		Gun Y_g =	
$X'-X_g$ =		$Y'-Y_g$ =	
Log $(X'-X_g)$ =		Log $(X'-X_g)$ =	
$-$Log $(Y'-Y_g)$ =		$-$Log sin α =	
Log tan α =		Log R_g =	
α =		R_g (Range) =	15,840 (15,800)
A_g (mils) =	366 FROM N		
(degrees) =	200.60 FROM S		

Zero (North) (South)

RANGE EFFECTS (use R_g as argument)	+	−
1. ROTATION Latitude of Gun = 42° N Azimuth of Target = 21° From N	27	
2. HEIGHT OF SITE Ht. of site − Tide = 430 feet − 10 feet = 420 feet	484	
3. WEIGHT OF PROJECTILES Average Wt. = 965 lbs. = 1 % (inc.) (dec.)	22	
4. MUZZLE VELOCITY Previously developed at 70° F = 2257 f.s. Change due to temperature of 62 °F = − 16 f.s. Corrected MV = 2241 f.s. Corrected MV − Standard MV = − 34 f.s.		376
5. WIND Speed of range component = − 8 m.p.h.		37
6. DENSITY Corr. dens. = $\left(\text{Ballistic density} - 0.3\frac{h}{100}\right)$ = 95 % − 1 % = 94 % Change = Corr. density − 100% = −6 %	262	
7. TEMPERATURE Temp. at battery = $\left(\text{temp. at m.d.p.} - \frac{1}{5}\frac{h}{100}\right)$ = 62 °F − 0 °F = 62 °F		6
TOTAL OF RANGE EFFECTS	795	419
RANGE CORRECTION = C_r		376
CORRECTED RANGE = $R_g + C_r - R_c$ =		15464

IND. GUN CORRECTIONS

	No. 1	No. 2	No. 3	No. 4
Corrected Range, R_c	15464	15464		
Corr. for calibration		− 55		
Corr. for displacement		+ 52		
Corr. for base ring		+ 16		
FIRING RANGE, R_f	15460	15480		
FIRING ELEVATION				

METEOROLOGICAL MESSAGE

Maximum ordinate	=	2,946 feet
Altitude of m.d.p.	=	200 feet
Temperature at m.d.p.	=	62 °F.
Azimuth of ballistic wind	=	6,000 mils
Speed of ballistic wind	=	11 m.p.h.
Ballistic density	=	95 %
Tide	=	+ 10 feet

θ = Az. of ballistic wind − A_g (in mils from N) = 5,630 mils

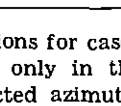
h = (Alt. Battery − Alt m.d.p.) = 400 feet − 200 feet = 200 feet

DEFLECTION EFFECTS (use R_g as argument)	R	L
1. ROTATION Latitude of Gun = 42° N Azimuth of Target = 21° From N	0.06	
2. WIND Speed of deflection component = − 8 m.p.h (R)	0.08	
3. DRIFT	0.30	
TOTAL OF DEFLECTION EFFECTS	0.44	
DEFLECTION CORRECTION = C_d		0.44
CORRECTED AZIMUTH = $A_g + C_d = A_c$ =		200.16

IND. GUN CORRECTIONS	No. 1	No. 2	No. 3	No. 4
Corrected Azimuth, A_c	200.16	200.16		
Corr. for calibration				
Corr. for displacement		+ 0.33		
FIRING AZIMUTH, A_f	200.16	200.49		

SYMBOLS

R_g = Map range of set-forward point
R_c = Corrected range for directing point
R_f = Firing range (corr. for ind. gun)
A_g = Map azimuth of set-forward point
A_c = Corrected azimuth for directing point
A_f = Firing azimuth (corr. for ind. gun)
C_r = Range correction
C_d = Deflection correction
θ = Chart direction of wind



Range difference = 106 sin(119.15° − 90°) = 52 Yds.

Parallax or Az. Difference = $57 \times \frac{106 \sin(180° - 119.15°)}{\text{Range from No 2}}$ =

$57 \times \frac{106 \sin 60.85°}{15840 + 52} = 0.33°$

Note: The multiplier 57 is a constant used in calculating Parallax. See end of par. 97.

b. Case II pointing.—Corrections for case II pointing differ from those given in *a* above only in the firing data for direction. Instead of a corrected azimuth for the set-forward point, a deflection from the position of the target at the instant of firing is computed. This deflection includes the expected travel of the target during the time of flight and corrections for drift, cross wind and, for long-range guns, rotation of the earth.

Given a battery of 155-mm guns, M1918MI (Firing Tables 155–B–4), firing normal charge, and using shell, HE Mk. III, fuze, short (Mk. IV*).

Corrected elevation: 402 mils.

Travel of the target during one observing interval (20 seconds): 1.35° (24 mils) to the right.

Cross wind: 6 miles per hour from the right.

The panoramic telescope M3 (with telescope mount M4) has been oriented so that the line of site is parallel with the axis of the bore when the deflection scale reading is 10.00°.

Required: Deflection for case II pointing.

Solution: Range corresponding to elevation of 402 mils= 12,200 yards. Time of flight=34.3 seconds.

	Deflection effects mils *Right*	*Left*
Cross wind		4
Drift	8	
Travel for 34 seconds		41
	—	—
	8	45
Deflection correction	37 mils=2.08°	
Deflection	10.00°+2.08°=12.08°	

CHAPTER 5

PROBLEMS RELATING TO POSITION

SECTION I

GENERAL

■ 47. DEFINITIONS.—*a. Mask.*—Any natural or artificial feature of or on the terrain which affords shelter from view.

b. Defilade.—The vertical distance by which a position is concealed from enemy observation. If the smoke and flash of firing are also concealed, the battery is said to have smoke and flash defilade.

c. Dead areas.—Areas that cannot be reached by fire. These may be caused by masks in front of the battery as well as by obstructions in the descending path of the projectile.

■ 48. LIMITATIONS OF FIRE DUE TO POSITION.—Local topography, both in vicinity of the firing position and in vicinity of the target, materially affects the possibilities of fire. If a selected position is defiladed to secure protection from enemy observation, the mask that furnishes the defilade may also serve to limit the minimum elevation at which the guns may be fired. Similarly, if there is a hill or an island in the field of fire, a certain portion of the area beyond the obstruction may be protected from fire.

SECTION II

MINIMUM ELEVATION

■ 49. GENERAL.—Figure 6 shows a gun G emplaced behind a mask H at a distance d from the gun. In order that the gun may shoot over the mask, it is necessary that it be pointed at a quadrant elevation greater than the quadrant elevation of the trajectory from *G* tangent to the crest at *H*. The usual amount added to the quadrant elevation for the tangent trajectory is two forks in elevation at the range *d*.

■ 50. DETERMINATION.—There are two methods used in determining the quadrant elevation for the tangent trajectory, depending on whether the mask is distant from or close to the gun position.

a. Distant mask.—If the mask is at such a range from the battery that the effect for target above gun (table C) may be found in the firing tables, the point *H* is treated as a target. The map range is corrected for height of site. (See pars. 23 to 26 incl.) Table A is then entered to determine the corresponding quadrant elevation. This elevation, increased

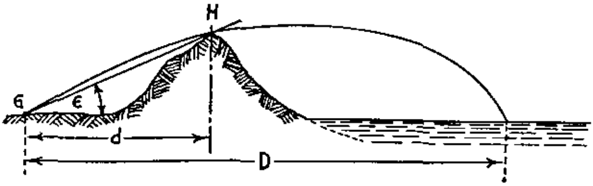


FIGURE 6.—Determination of minimum elevation.

by two forks at the range *d*, is the minimum quadrant elevation.

b. Near mask.—If the mask is too close to obtain the correction for target above gun from the firing tables, the angle of site (ϵ) is determined. This angle is added to the quadrant elevation plus two forks for the range *d* (computed from the probable error in yards) to give the minimum quadrant elevation.

■ 51. EXAMPLES.—*a.* A battery of 12-inch guns firing 975-pound A.P. projectile (FT 12-F-3) is to be fired over an island 8,000 yards from the battery position. The top of the island is 450 feet higher than the battery position. What is the minimum quadrant elevation at which the guns can be fired with assurance of clearing the island?

Solution:

	Yards
Range to crest of island	8,000
Correction for height of site (table C)	1,396
Corrected range	9,396
Corrected elevation (from table A)	6°09′
Add 2 forks at 8,000 yards	18′
Minimum quadrant elevation	6°27′

NOTE.—The value of one fork in elevation at 8,000 yards is obtained from table A by converting one fork in range to a fork in elevation by means of the data in columns 5 and 16; for example, 1 fork (or 4 P. E.'s) = 204 yards and is equivalent to $2.04 \times 4.5' = 9.2'$.

b. The same battery is located behind a sand dune whose crest is 40 feet above the guns. If the dune is 300 yards from the guns, what is the minimum elevation at which the guns can be fired?

Solution:

Tangent, angle of site=13/300	0.043333
Angle of site	2°29′
Elevation for 300 yards (table A)	11′
Add two forks (table A, col. 5 and 16)	07′
Minimum elevation	2°47′

c. The following example is given to illustrate this type of problem, using firing tables such as 155–B–4, which do not give the effect of height of site in yards or range, but which permit correction for height of site by means of a corrected angle of site:

A battery of 155-mm guns using normal charge and firing shell, HE, Mk. III, with fuze Mk. IV* (Firing Tables 155–B–4), is to be fired over an island 6,000 yards from the battery position. The top of the island is 1,000 feet higher than the battery position. What is the minimum quadrant elevation at which the guns can be fired with assurance of clearing the island?

Solution:

Map range to crest of island	6,000 yards.
Difference in altitude, target above guns.	1,000 feet or 333 yards.
Angle of site	333/6,000 or +56 mils.
Correction, angle of site, for non-rigidity of trajectory	+1.1 mils.
(Column 13, table A, part 2b–1 of firing tables.	56×+.02 mils.

	Mils
Corrected angle of site=56+1.1	57.1
Quadrant angle of elevation for 6,000 yards	123.6
Corrected angle of elevation	180.7
Add 2 forks at 6,000 yds. (columns 4 and 7, table A)	7.4
Minimum elevation	188

SECTION III

MINIMUM RANGE

■ 52. GENERAL.—Frequently it is of importance to know the minimum range at which a gun can fire. From figure 6 it is obvious that the minimum range is mostly dependent on the minimum elevation at which the gun can be laid. However, minimum elevation fixes only the range to the level point. That range must be corrected for height of site to give the minimum range. The correction for height of site may be determined approximately by use of the slope of fall listed in table A of the firing tables. If more accuracy is needed, the problem may be solved by computation from the firing tables, using the method given in paragraph ~~20.~~ 26 B

■ 53. DETERMINATION.—*a. Approximate solution.*—Using the minimum elevation as determined by one of the methods above, extract from table A of the firing tables the corresponding slope of fall which is given in the general form, 1 on n. This determines the slope of the line BS in figure 7. Then, by simple proportion, $BB'/B'S=1/n$ and $B'S=n\times BB'$. The approximate range to the splash S may then be computed by the formula: Expected range=$GB+B'S$. Since this method is based on the assumption that the trajectory is a straight line beyond the level point, the approximation will be close only when the angle of fall is large.

b. Computation from firing tables.—This method consists of determining, by use of table B (or C), the range which, when corrected for the height of site, equals the range to the level point as determined by the minimum elevation.

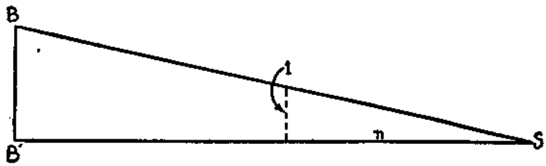


FIGURE 7.—Minimum range by approximate solution.

(See par. 26*b*.) If table B does not list effects for the given height of site at sufficiently short ranges, extrapolation may be necessary. An example of the procedure in this case is found in paragraph 56.

■ 54. EXAMPLES.—A battery of 12-inch guns firing 975-pound A. P. projectile (Firing Tables 12–F–3) is so emplaced that its minimum elevation is 2°59′. The altitude of the battery is 60 feet above sea level. What is the minimum range at which the battery can fire at a naval target?

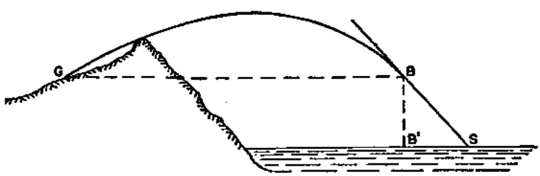


FIGURE 8.—Illustration for example, paragraph 54.

a. Approximate solution.—The following data are extracted from table A:

Minimum elevation	2° 59′.
Level point range	5,000 yards.
Slope of fall	1 on 18.

The expected range is therefore 5,000+(18×60/3)=5,360 yards.

b. Computation from firing tables.—As determined above, the range to the level point is 5,000 yards. Enter table B of part 2 with a height of site of 60 feet as an argument. It is known that the range to the splash will be greater than 5,000 yards, and that the range minus the effect of height of site must equal 5,000 yards. It can be seen from the following that the correct value lies between 5,000 and 5,500 yards.

Map range	5,000	R	5,500
Height of site correction	−354	$-\Delta R$	−315
Level point range	4,646	5,000	5,185

$$\therefore R = 5{,}000 + \frac{5000-4646}{5185-4646} \times 500 = 5{,}328 \text{ yards}$$

SECTION IV

DEAD AREA CHART

■ 55. GENERAL.—Dead areas are those areas that cannot be reached by fire. They may be due to masks, to high points in the field of fire, or to limitation in the traverse of the gun. If a battery is emplaced on an elevation, there may be water areas inside the minimum range which cannot be reached by fire. If all dead areas are plotted on a map, the possibilities of fire may be studied therefrom. A dead area chart is ordinarily constructed by drawing rays from the battery at convenient intervals and determining the portions of each ray which cannot be reached by fire from the battery. The ends of these portions are then connected by lines which inclose the various dead areas included in the battery field of fire (fig. 9).

■ 56. EXAMPLES.—A portion of a dead area chart is shown in figure 9. This chart was constructed for a battery of 12-inch seacoast guns, M1895, on barbette carriage, M1917, firing the 975-pound projectile (FT 12–F–3). The height of site of the battery is 200 feet and the minimum elevation is 0°. An island is shown between the 180° and 200° azimuth rays. Its crest is 400 feet high and is 3,000 yards from the battery. Another island is shown at 260° azimuth; its highest point is 600 feet above sea level and is 15,000 yards from the battery. Procedure for constructing the chart is as follows:

a. Plot of battery position and azimuth rays within field of fire.—Mark the position of the battery and draw through it a line to represent the center line of the field of fire. With

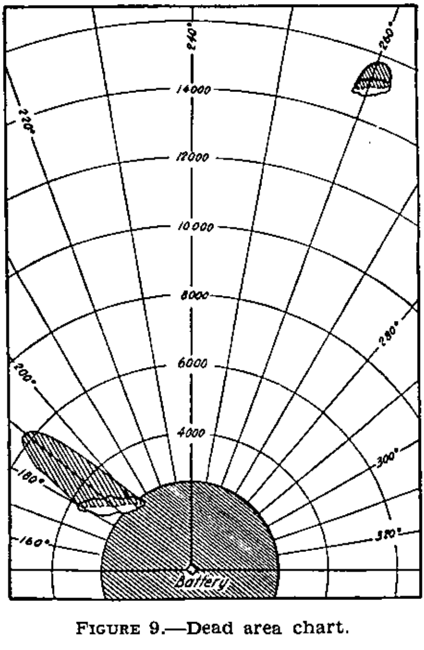


FIGURE 9.—Dead area chart.

this line as a reference line, construct rays of azimuth for each 10° within the field of fire. In the figure, it was assumed that the azimuth of the center line is 240° and that the field of fire extends through 90° on each side of that line.

b. Plot of arcs of range from minimum to maximum.—(1) *Minimum range.*—The minimum elevation is given as 0°. The corresponding level point range is 0 yards. The problem is to find a range for which the correction for height of site exactly equals the range. In table B, part 2, it is found by inspection that the range is less than 3,000 yards, from which the upper limit of the interpolating interval of level point ranges may be determined. Before the lower limit can be determined, the value of the range effect for a range of 2,500 yards must be computed. There are two ways in which this value might be extrapolated, either vertically in the 2,500-yard column or horizontally in the 200-foot row. More accurate results will be obtained from the former unless second and third order differences are considered. This effect, when found and applied to the range of 2,500 yards, will give a negative range to the level point, which is the lower limit of the interpolating interval. From here on the procedure is similar to that for a range within the tabulated values. The computations are as follows:

Ranges	2,500	3,000
Range effects	−2,673	−2,179
Level point ranges	−173	+821

Map range=2,500+(173×500/994)=2,587 yards.

Map range	2,500	R	3,000
Height of site correction	−2,673	$-\Delta R$	−2,179
Level point range	−173	0	821

$$R=2{,}500+\left(\frac{173}{173+821}\times 500\right)=2{,}587\text{ yards.}$$

The approximate minimum range is therefore 2,590 yards, and an arc of this radius may be drawn about the battery position.

(2) *Maximum range.*—From "General Information" the section in the firing tables we find that the maximum elevation of this type of gun is 35°. This gives a range to the level point of 29,310 yards (table A). From table B we find the map range for a height of target of −200 feet to be approximately 24,370 yards. Arcs for each 2,000 yards of range between the limiting ranges are usually shown on the chart.

c. Plot of dead areas in the field of fire.—Obstructions such as the two islands are drawn in to scale on the figure and the dead areas computed and sketched in. Dead areas are usually shaded to distinguish them from clear areas.

In figure 9, the minimum ranges to the outer limits of the dead areas were determined for the midpoints of the islands and the outlines of those areas sketched in approximately. The computations were made as described in paragraphs 50 and 53.

(1) *Island at 3,000 yards range.*—This island is 400 feet high. It is assumed that several observing stations are located on this island and that the minimum elevation for the tangent trajectory must be increased by two forks to insure clearing the crest.

	Yards
Range to crest	3,000
Range correction (table C)	2,038
Corrected range	5,038
Corrected elevation (table A)	3°01′
Add two forks at 3,000 yards (see note)	10′
Minimum elevation	3°11′
	Yards
Level point range for 3°11′ (table A)	5,300
Minimum range corrected for height of site (table B)	6,220

NOTE.—The value of one fork in elevation at 3,000 yards is obtained from table A by converting one fork in range to a fork in elevation by means of the data in columns 4 and 16; for example, 1 fork (or 4 P. E.'s) = 144 yards = $1.44 \times 3.6 = 5.2'$.

(2) *Island at 15,000 yards.*—This island is 600 feet high. It is assumed that it is unoccupied and that the minimum elevation of the tangent trajectory may be used in computing the minimum range.

	Yards
Range to crest	15,000
Range correction (table C)	494
Level point range	15,494
Minimum range corrected for height of site (table B)	15,730

CHAPTER 6

ACCURACY OF POSITION-FINDING METHODS

SECTION I

GENERAL

■ 57. GENERAL.—Necessity for an understanding of the capabilities and limitations of any position-finding system is obvious. By careful consideration of the various sources of error in determining position data, it is possible to reduce the inaccuracies to a minimum thereby increasing the effectiveness of fire. The purpose of this chapter is to analyze the elements of personnel and matériel in each position-finding system with a view to assisting an artillery commander to obtain the best results from the resources at his command.

■ 58. CURVATURE AND REFRACTION.—*a*. Atmospheric refraction and the earth's curvature are directly associated with observing methods. The effect of curvature of the earth is to limit the range of observation and to cause the actual location of the target to be lowered with respect to the observer. Atmospheric refraction causes the rays of light passing from the target to the observer to be bent downward, increasing the field of view, and causing the target apparently to be raised from its true position. This effect is variable, being generally greatest during the night and least during the day. The apparent vertical displacement of the target due to refraction in observation partially counteracts the lowering of the target due to curvature of the earth.

b. The measure of refraction, called the "coefficient of refraction," is represented by the letter m and is the ratio between the vertical angle by which the observed point is apparently displaced and the angle at the center of the earth subtended by the range. Along the coast on a typical day the value of m from 10:00 AM to 3:00 PM is nearly constant and at a minimum, its value being approximately 0.0714. At other hours the value is greater, the mean for a typical 24-hour period being about 0.083.

c. The combined vertical effect of refraction and curvature is of considerable magnitude. Therefore a knowledge of this effect is essential in the solution of practical problems, such as determining the height of site required for an observing station or calculating the range of observation of an observing station already constructed. Table I in appendix VI gives the combined vertical effect h in feet for ranges up to 50,000

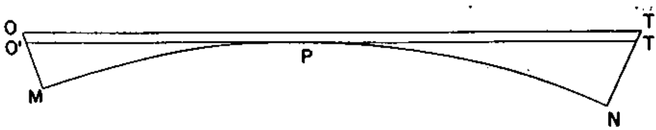


FIGURE 10.—Determination of required height of instrument.

yards. In the absence of a table, the following equation will give fairly accurate results:

$$h \text{ (in feet)} = 0.18 \times (\text{range in thousands of yards})^2$$

Example: It is desired to determine the height of instrument necessary to track targets at a range of 30,000 yards from the observation station, assuming that the height of the point to be observed on the target is 55 feet above the water. (See fig. 10.)

Solution: For tracking, the line of sight should be at least 5 feet above the water. Therefore 5 feet is subtracted from TN, the height of the point to be observed. Entering table I, with the argument h equal to 50 feet, the distance to the horizon, $T'P$, is found by interpolation to be 16,400 yards. The distance $O'P$ is 30,000—16,400 or 13,600 yards. From table I, the distance $O'M$ is found to be 34 feet. Adding 5 feet to raise the line of sight above the water that amount, the proper height of instrument is 34+5=39 feet.

SECTION II

HORIZONTAL BASE SYSTEM

■ 59. BASE LINES.—*a*. Since the position of the target is obtained from the solution of a triangle of which one side is the base line of the observing system, it is essential that the length and azimuth of this line be determined to within the smallest limits capable of being set on the instruments used in position finding. The orientation data given in emplacement books, having been computed from measurements made with care and deliberation, are usually listed to the nearest tenth of a yard and the nearest thousandth of a degree. However, the accuracy of position-finding instruments does not warrant such precise calculations for tests which a battery commander may wish to make of his system; results to the nearest yard and the nearest hundredth of a degree (or minute of bearing) are sufficiently accurate for this purpose. They may be made with four-place logarithm tables but the use of five-place tables will eliminate some interpolation. If the calculations are made on a machine, natural functions of angles to four decimal places may be used.

b. Accuracy of ranges obtained by the horizontal base system is directly affected by the size of the angle formed at the target by lines of sight from the two base ends. Both for the required geometrical accuracy and the satisfactory mechanical operation of plotting boards, this angle should not be less than 15°. It being impracticable to increase the length of the base line in order to prevent the angle ever becoming less than 15°, the difficulty may be minimized by the selection of a base line whose perpendicular bisector will pass through the center of the area which it is desired to cover.

■ 60. OBSERVATION.—*a*. In the horizontal base position-finding system, azimuths of the target from the base end stations are obtained by the use of azimuth-measuring instruments. Accuracy of this determination of azimuths is dependent upon the skill of the observer, his physical condition, visibility conditions, size of aiming point, and mechanical condition of the instrument. (A complete discussion of the effect of small errors in observed azimuths is found in appendix III.)

b. The observer furnishes the most important source of inaccuracy. Errors contributed by him may be due to his failure to remove parallax entirely from his instrument, his failing to orient the instrument accurately, and his lack of skill in having the vertical cross hair precisely on the target at the instant for which a reading is to be taken. Thorough training of an observer will reduce these errors to a minimum.

c. A check on the general accuracy of an observer may be obtained by examining a record of his readings on a target moving at constant speed on a straight line. Over short periods of time, the angular travel of the target will be nearly uniform. Therefore the first order of differences of the readings should not vary greatly during short periods of time, those of a well-trained observer seldom varying more than about .01°.

SECTION III

NOTES ON ACCURACY OF OBSERVATION APPLICABLE TO RANGE FINDING BY BOTH SELF-CONTAINED AND VERTICAL BASE SYSTEMS

■ 61. SOLUTION OF TRIANGLE BY SHORT BASE LINE.—In the self-contained and vertical base systems of position finding, the range to the target is determined by the solution of a triangle, the base of which is very short in comparison with the other sides of the triangle. Therefore, an extremely accurate measurement of the angles adjacent to the base line is required.

■ 62. ERRORS IN OBSERVED RANGES.—*a*. It will be found in actual practice that due to limitations of the human eye, an observer cannot read ranges without error. If an observer makes ten readings on a fixed datum point at a range suitable for the instrument and the readings are tabulated, it will be found that the readings vary and the mean of the ten readings may vary from the known range to the datum point. Such results indicate the presence of two kinds of errors; systematic errors and accidental errors. The systematic error is one that is present in all the readings due to faulty adjustment of the instrument. For this reason, it is sometimes known as the instrumental error. It will be the difference between the known range to the datum point and the mean of a series of many readings; in this instance, the mean of the ten readings taken. It may be practically eliminated by making the prescribed range adjustment to fit the instrument to the observer in the case of a self-contained range finder and by proper height of site and refraction adjustment in the case of a depression position finder. The accidental error is that error present in an individual reading only and is equal to the difference between the individual reading and the mean of all other readings. It is caused by imperceptible errors in gaining coincidence or stereoscopic contact or errors in laying the horizontal cross wire on the water line of the target. Accidental errors cannot be entirely eliminated but may be reduced by proper training of the observer.

b. Under excellent conditions of visibility, the smallest angle distinguishable by the naked eye of a well-trained observer may be considered to be about 40 seconds of arc. For example, an observer using a depression position finder that has no magnifying power would be incapable of distinguishing a lack of coincidence between the horizontal cross wire and the water line of a stationary target of less than approximately 40 seconds of arc. However, the theory of errors applies, and the observer would make a mean angular error of only about 12 seconds of arc in a long series of readings. (See pars. 107 and 108.)

NOTE.—The effect of magnification is to make smaller angles distinguishable, the distinguishable angle theoretically varying inversely as the power of the optical system. Practically, the distinguishable angle does not vary strictly inversely to the power, but for purposes of comparative analyses it is accepted as such in this manual. Thus the error of any individual setting of the instrument is taken as the angular error of the observer's eye divided by the magnifying power. A magnification of about 30 power has been found to be the maximum that can be used. Greater magnification causes dullness of the target image and increases the effect of poor atmospheric conditions and vibrations. For use under conditions of poor visibility, a choice of lower magnification is furnished on instruments.

c. The mean angular error of 12 seconds of arc given above is considered the smallest value that can be established by a well-trained observer, under perfect conditions of visibility, with a stationary target. With observers of less ability, with a moving target, and under poorer conditions of visibility, this angle may be as much as 60 seconds, or possibly more. In the vertical base system, the rise and fall of the water line on the target due to rough seas may cause errors of many seconds of arc. Therefore, one should not be surprised to find in actual practice that the mean error of an observer corresponds to an angle considerably greater than 12 seconds of arc.

■ 63. Accuracy Index of an Observer.—Since there is a great difference in the inherent capabilities of observers, it is found of great convenience to establish what is known as the accuracy index of an observer. Represented by the symbol $\Delta\alpha'$, this is the mean angular error, in seconds of arc, made by the observer in a series of readings with a magnification of 1 (the naked eye). By calculating the accuracy index of several observers, a comparison of their efficiency can be made regardless of variations in range, height of site, or magnifying power, provided that the tests made to establish the indices are conducted under the same visibility conditions; that is, at the same time and in the same general locality. The general procedure followed in calculating the accuracy index of an observer is to have him make a series of readings, not less than ten, on a point at a known range, and from the tabulated readings determine the arithmetic mean of the accidental errors in yards of range. This arithmetic mean is then converted into a corresponding angular error by means of the proper equation for the type of instrument. (The equations referred to are discussed in the sections under the particular kind of system used. The derivation of the equations and determination of the effect of small angular errors are discussed in appendices IV and V.)

Section IV

OBSERVATION WITH COINCIDENT AND STEREOSCOPIC RANGE FINDERS

■ 64. General.—In effect, the coincident or stereoscopic range finder determines the range to a target by solving the equation

$$\tan \alpha = \frac{b}{R}$$

in which α is the parallax angle between the lines of sight from the target to the penta prism at each end of the instrument, b is the length in yards between the penta prisms, and R is the range to the target in yards. A small error in the measurement of the angle α obviously will make an appreciable error in the determination of R. An angular error of $\Delta\alpha$ seconds of arc will make an error of ΔR yards of range as follows:

$$\Delta R = -\frac{R^2 \times \Delta\alpha}{b \times M \times 206{,}000} \qquad (1)$$

In this equation, M is the magnification of the instrument and the term 206,000 is used to permit the use of $\Delta\alpha$ in seconds of arc rather than in radians.

■ 65. FACTORS AFFECTING ACCURACY.—The accuracy of self-contained range finders, regardless of type, depends principally upon three factors: effective power of the instrument, ability of observer to determine coincidence or stereoscopic contact, and visibility.

a. The effective power of the instrument is defined as the product of the length times the magnification. It will be noted in the equation above that the effective power is in the denominator of the right-hand member, and therefore the greater the effective power of the instrument the smaller will be the resulting range error. Other things being equal, a 30-foot range finder should determine ranges with errors one-half the size of those of a 15-foot instrument of the same magnification. Theoretically, the same principle is true with magnifying power, but practically it is not always so. The observer's accuracy depends on the brightness and sharpness of the target image. Brightness decreases with an increase in magnification while the effect of heat waves, poor visibility, and vibration becomes greater. It is possible therefore to use higher magnifying power only when sharp definition will permit.

b. The ability of an observer to determine coincidence (or stereoscopic contact) precisely depends primarily on his eyesight and his training, although such factors as whether he is sick, tired, or nervous will cause variations in his ability. The limitations of the eye, varying in different observers, cause unavoidable accidental errors but it has been found that by constant training these errors can be considerably reduced. Present beliefs indicate that an observer, especially one using a stereoscopic range finder, in order to maintain a high state of efficiency must have at least 1 hour of training daily 3 or 4 days a week. With such training, the average angular error should be between 12 and 17 seconds of arc with a stationary target and under conditions of excellent visibility. For a moving target, the expected error may be doubled at least.

c. Visibility plays an important part in range finding by self-contained instruments. When the air is "boiling," the irregular refraction makes observation very difficult. When observing at night with stereoscopic range finders, it will be found that the difference in amount of illumination of the target and the reticle symbols increases the difficulty of accurate observation.

■ 66. TEST OF ACCURACY OF AN OBSERVER.—Assume that it is desired to determine the accuracy index of an observer using a self-contained range finder. Equation (1) above may be used, letting the arithmetical mean of the accidental errors be represented by ΔR and substituting $\Delta a'$ for Δa.

Example: Observations were taken with a 15-foot coincidence range finder of 28-power magnification on a datum point at 9,760 yards range. Tabulation of the results is as follows:

Reading No.	Actual range	Observed range	Mean of observed ranges	Accidental error
	Yards	*Yards*	*Yards*	*Yards*
1	9,760	9,660	9,510	150
2	9,760	9,600	9,510	90
3	9,760	9,520	9,510	10
4	9,760	9,450	9,510	60
5	9,760	9,550	9,510	40
6	9,760	9,400	9,510	110
7	9,760	9,530	9,510	20
8	9,760	9,540	9,510	30
9	9,760	9,430	9,510	80
10	9,760	9,420	9,510	90
Mean	9,760	9,510		68

This indicates a systematic error of 250 yards (the amount by which the instrument should be adjusted for present atmospheric conditions and to fit it to the individual observer) and a mean accidental error of 68 yards. Rearranging equation (1)—

$$\Delta\alpha' = \frac{\Delta R \times b \times M \times 206{,}000}{R^2}$$

$b = 5$ yards $\quad \Delta R = 68$ yards
$M = 28 \quad R = 9{,}760$ yards
By slide rule, $\Delta\alpha' = 21$ seconds of arc

SECTION V

OBSERVATION WITH DEPRESSION POSITION-FINDER

■ 67. EFFECTIVE HEIGHT.—*a*. The combined vertical effect of curvature of the earth and atmospheric refraction acts to reduce the effective base of a depression position-finder as the range increases.

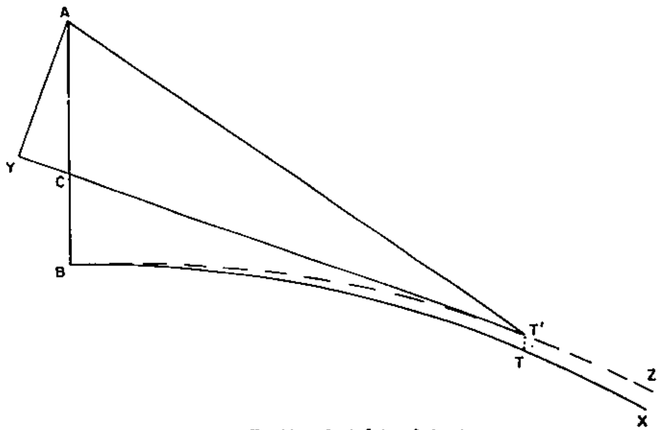


FIGURE 11.—Effective height of instrument.

This is illustrated in figure 11, in which *XTB* is the earth's surface; *ZT'B* is the apparent surface of the earth, or the surface as it would appear, due to refraction, to an observer with an instrument anywhere on line *AB* (note below); *T* represents a target; and the line $T'Y$ is the tangent to the apparent surface of the earth at T'. Obviously, to a person at C, the height CB is of no assistance for determining whether the target is at T or farther or nearer than T. It will appear to be on the horizon. The surface of the sea for 2,000 or 3,000 yards on either side of T may be considered as flat and in the same plane as the tangent $T'C$. Therefore to obtain a range determination of T, it is necessary to locate the observing instrument above C and if one is placed at A, the range finding triangle is $AT'Y$, AY being at right angles to $T'Y$. Then AC (for all practical purposes it is equal to AY) is called the "effective height" of the instrument. It is equal to the height of instrument above sea level minus the vertical effect of curvature and refraction, or as represented by symbols, equals $b-h$. It will be noted that as the range increases, h also increases. When the target gets so distant that it is actually on the apparent horizon, h is equal to b, no effective height remains, and range cannot be determined.

NOTE.—The vertical effect of refraction is about 1/6 that of curvature. Their combined effect then is about 5/6 the effect of curvature.

b. From figure 11, it is obvious that as the range increases and the effective height of the instrument decreases, the accuracy of range finding decreases rapidly. This fact is illustrated mathematically in the following equation in which ΔR is the range error in yards caused by an observer's angular error of $\Delta\alpha$ seconds of arc in water lining the target: (See appendix IV for complete derivation of the formula.)

$$\Delta R = -\frac{R^2 \times \Delta\alpha}{(b-h)\times M \times 206{,}000} \tag{2}$$

(The similarity between this equation and equation (1) of paragraph 64 may be noted, in that $(b-h)$ replaces b.)

c. The change in range reading caused by a small change in effective height in any instrument is considerable (appendix IV). Since the effective height is constantly changing due to varying tide and refraction, it becomes imperative that an observer make frequent test readings on known datum points and readjust the height setting of his instrument. At least three datum points, one each at short, medium, and near maximum range should be available, and should be objects on which the water line is clearly defined. The datum points may be water-borne objects, such as buoys, or fixed objects on which the water always registers.

■ 68. FACTORS AFFECTING ACCURACY OF OBSERVATION.—*a*. The greater the magnification of the telescope, *provided the water line is well defined*, the greater the accuracy that can be obtained. However, good definition is of most importance, and the highest power eyepiece may be used only when atmospheric conditions permit.

b. The roughness of the sea has considerable effect on accuracy, causing a continual variation in the location of the water line on a target. An endeavor should be made to lay the horizontal wire midway between the highest and lowest position of the water line.

c. The thickness of the cross wire may cause a considerable error in water lining. Therefore, the upper edge of the horizontal wire should be used when adjusting the instrument on a datum point and following a target. To accomplish this, the cross wire should be brought to the water line by a movement in elevation.

d. If the target image is not precisely in the plane of the cross wires, that is, if parallax is not removed by correctly focusing the instrument, enormous errors in range readings will result; the errors being sometimes plus and sometimes minus, depending upon the position of the observer's eye.

■ 69. TEST OF ACCURACY OF AN OBSERVER.—Previous discussion of the limitations of an observer's eye applies to observers using a depression position finder as well as to those using self-contained range finders. The selection of observers may well be based upon the accuracy index exhibited by each observer in taking long series of readings on targets at a known range. The accuracy index of an observer may be calculated by the use of equation (2) above.

Example: Observations were taken using a 25-power D. P. F. having a height of instrument of 120 feet, on a moving target. Tabulation of results is as follows:

Reading No.	True range (by horizontal base)	D. P. F. range	Error D. P. F.	Systematic error	Accidental error
	Yards	*Yards*	*Yards*	*Yards*	*Yards*
1	10,480	10,520	+40	+18	22
2	10,380	10,450	+70	+18	52
3	10,270	10,300	+30	+18	12
4	10,160	10,140	−20	+18	38
5	10,050	10,020	−30	+18	48
6	9,940	9,950	+10	+18	3
7	9,830	9,880	+50	+18	82
8	9,730	9,730	0	+18	18
9	9,630	9,590	−40	+18	58
10	9,530	9,600	+70	+18	52
Mean	10,000	10,018			34

Systematic error of instrument=+18 yards. Average accidental error of observer=34 yards. What is the accuracy index of the observer?

Solution:

$$\Delta\alpha' = \frac{\Delta R \times (b-h) \times M \times 206{,}000}{R^2}$$

$M=25$ .... $b=40$ yards.
$R=10{,}000$ yards (note below) .. $h=6$ yards (from table I).
$\Delta R=34$ yards .... $b-h=34$ yards.
By slide rule .... $\Delta\alpha'=59.6$ or 60 sec.

Since the linear range error corresponding to a definite angular error varies nearly in proportion to the square of the range, in cases where the range to the target changes rapidly the readings should be grouped into sets, each covering only a moderate change in range. The angular error of each set is then calculated and the mean is accepted as the observer's accuracy index.

NOTE.—The value of R for use in the formula and table I should be the mean true range for each series of about ten readings.

SECTION VI

PLOTTING BOARDS

■ 70. GENERAL.—Use is made of the plotting board to solve a triangulation problem to locate observed positions of the target and to relocate so that necessary data are referred to the directing point of the battery. These being mechanical operations, it is desirable that the board perform them accurately.

■ 71. CHECK POINTS.—In order to check the mechanical accuracy of a plotting board, it becomes necessary to have certain points in the field of fire for which the accurate range and azimuth are known from the directing point of the battery and from each observation station. These points are known as check points. They should be so located that they provide a check for each part of the field of fire at normal ranges. For example, a battery of 12-inch guns mounted on barbette carriages with a field of fire of 150° should have a minimum of three check points at about 18,000 yards range, one each for left, center, and right field of fire and a minimum of two check points for the area normally used for subcaliber firing. There are two situations found in computing check points; first, when the coordinates of points suitable for use as check points are known, and second, when the coordinates are not known.

■ 72. COMPUTATION OF CHECK POINTS, COORDINATES KNOWN.—When the coordinates of check points, directing point, and observing stations are known, the range and azimuth between any two of these points may be easily calculated. The following equations may be used:

Tangent bearing (angle line between points makes with Y-axis) $=\Delta x/\Delta y$.

Azimuth, a function of the bearing, depending on the quadrant.

Range$=\Delta x/\sin$ bearing$=\Delta y/\cos$ bearing.

NOTE.—Δy should be corrected for magnification of scale when using standard grid coordinates. (See table K, TM 4–225 (now published as TM 2160–25).)

■ 73. COMPUTATION OF CHECK POINTS, COORDINATES UNKNOWN.—When coordinates of elements of the battery are not known, it will be necessary to select points in the correct areas and make required calculations from assumed values of azimuth from each end of the base line or of range and azimuth from the directing point. A suggested method of obtaining basic data for computations is to choose a point on the plotting board, bring the station arms up to it, and read the azimuths. Solution of the triangles may be made by use of the following equations (fig. 12):

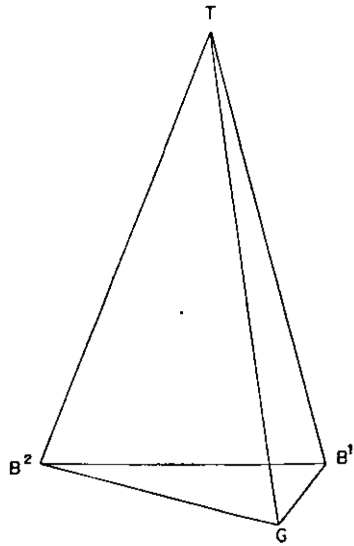


FIGURE 12.—Location of target by horizontal base.

a. Range and azimuth from observing stations.

$$\text{Angle } TB^1B^2 = \pm\text{Azimuth } B^1T \mp \text{Azimuth } B^1B^2 \tag{1}$$
$$\text{Angle } TB^2B^1 = \pm\text{Azimuth } B^2B^1 \mp \text{Azimuth } B^2T \tag{2}$$
$$\text{Angle } B^1TB^2 = 180° - (TB^1B^2 + TB^2B^1) \tag{3}$$
$$\text{Range } B^1T = \frac{B^1B^2 \sin TB^2B^1}{\sin B^1TB^2} \tag{4}$$
$$\text{Range } B^2T = \frac{B^1B^2 \sin TB^1B^2}{\sin B^1TB^2} \tag{5}$$

It is obvious that in order to use the above equations it will be necessary to draw a sketch of the situation and consult it carefully in solving for the various angles.

b. Range and azimuth from directing point.

$$\text{Angle } GB^2T = \pm \text{Azimuth } B^2G \mp \text{Azimuth } B^2T \qquad (1)$$

$$\text{Tan } \tfrac{1}{2}\,(B^2GT - B^2TG) = \frac{B^2T - GB^2}{B^2T + GB^2} \times \tan \tfrac{1}{2}(180° - GB^2T) \qquad (2)$$

$$\tfrac{1}{2}(B^2GT + B^2TG) = \tfrac{1}{2}(180° - GB^2T) \qquad (3)$$

$$\text{Angle } B^2GT = \tfrac{1}{2}(B^2GT + B^2TG) + \tfrac{1}{2}(B^2GT - B^2TG) \qquad (4)$$

$$\text{Azimuth } GT = \text{Azimuth } GB^2 \pm \text{Angle } B^2GT \qquad (5)$$

$$\text{Range } GT = \frac{B^2T \sin GB^2T}{\sin B^2GT} \qquad (6)$$

NOTE.—A similar set of equations may be derived for use with B^1T and B^1GT. The station more distant from G should be used in the solution because it employs the larger angle at T.

■ 74. EXAMPLES.—*a*. Given the following data (standard grid coordinates):

Point:	X	Y
B^1	679, 622	1, 582, 905
B^2	672, 134. 9	1, 584, 273
G	678, 979. 8	1, 582, 898
T	676, 632	1, 602, 408

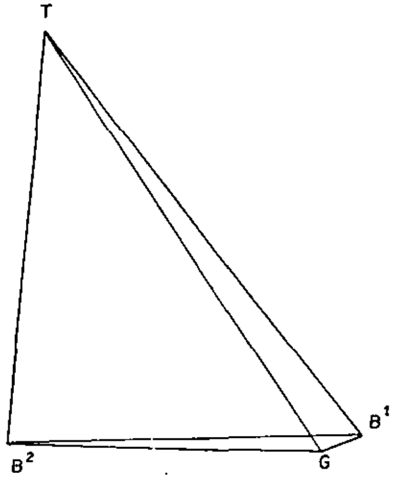


FIGURE 13.—Situation sketch for example, paragraph 74*a*.

Latitude and longitude of directing point *G*, 37° *N*. and 76°18′ *W*., respectively. Correction to Δy for magnification of scale error=1.07 yards per thousand yards. (See TM 4–225 (now published as TM 2160–25).)

Compute the azimuth (zero *S*) and range from the observing stations and the directing point to the check point *T*.

Solution (fig. 13):

	X Yards		Y Yards
B^1	679,622		1,582,905
T	676,632		1,602,408
Δx	2,990	Δy (uncorrected)	19,503
		correction	−21
		Δy (corrected)	19,482

	X Yards		Y Yards
B^2	672, 135		1, 584, 273
T	676, 632		1, 602, 408
Δx	4, 497	Δy (uncorrected)	18, 135
		correction	−19
		Δy (corrected)	18, 116
G	678, 979. 8		1, 582, 898
T	676, 632		1, 602, 408
Δx	2, 347. 8	Δy (uncorrected)	19, 510
		correction	−21
		Δy (corrected)	19, 489

$$\text{Tan bearing } B^1T = \frac{2990}{19482}$$

$$\log 2{,}990 = 3.47567$$
$$\log 19{,}482 = 4.28963$$

$$\log \tan \text{bearing} = 9.18604 - 10$$
$$\text{Bearing } B^1T = 8°44′\ (8.73°)$$
$$\text{Azimuth } B^1T = 180.00 - 8.73$$
$$= 171.27°$$

$$\text{Tan bearing } B^2T = \frac{4497}{18116}$$

$$\log 4{,}497 = 3.65292$$
$$\log 18{,}116 = 4.25806$$
$$\log \tan \text{bearing} = 9.39486 - 10$$
$$\text{Bearing } B^2T = 13°56' \ (13.93°)$$
$$\text{Azimuth } B^2T = 180.00 - 13.93$$
$$= 193.93°$$
$$\text{Tan bearing } GT = \frac{2348}{19489}$$
$$\log 2{,}348 = 3.37070$$
$$\log 19{,}489 = 4.28979$$
$$\log \tan \text{bearing} = 9.08091 - 10$$
$$\text{Bearing } GT = 6°52' \ (6.87°)$$
$$\text{Azimuth } GT = 180.00 - 6.87$$
$$= 173.13°$$

$$\text{Range } B^1T = \frac{2990}{\sin 8°44'}$$
$$\log 2{,}990 = 3.47567$$
$$\log \sin 8°44' = 9.18135 - 10$$
$$\log \text{range} = 4.29432$$
$$\text{Range } B^1T = 19{,}693 \text{ yards}$$

$$\text{Range } B^2T = \frac{4497}{\sin 13°56'}$$
$$\log 4{,}497 = 3.65292$$
$$\log \sin 13°56' = 9.38164 - 10$$
$$\log \text{range} = 4.27128$$
$$\text{Range } B^2T = 18{,}666 \text{ yards}$$

$$\text{Range } GT = \frac{2{,}348}{\sin 6°52'}$$
$$\log 2{,}348 = 3.37070$$
$$\log \sin 6°52' = 9.07756 - 10$$
$$\log \text{range} = 4.29314$$
$$\text{Range } GT = 19{,}640 \text{ yards}$$

b. An 8-inch railway battery is emplaced outside of a harbor defense where accurate grid coordinates of elements of the battery are not known. The following orientation data are determined by survey:

	Distance (yards)	*Azimuth (zero south) (degrees)*
B^1 to B^2	3,432.3	280.09
G to B^1	275.6	151.87
G to B^2	3,191.2	276.24

Assume check point No. 2 (T) such that—

Azimuth B^1 to T is 31.00°
Azimuth B^2 to T is 45.00°

Compute the data for checking the orientation of the plotting board on the point T.

Solution (fig. 14):

$$\begin{aligned}
&\text{Angle } TB^1B^2 = \text{Azimuth } B^1T - \text{Azimuth } B^1B^2 = 110.91^\circ\\
&\text{Angle } TB^2B^1 = \text{Azimuth } B^2B^1 - \text{Azimuth } B^2T = 55.09^\circ\\
&\text{Angle } B^1TB^2 = 180^\circ - (110.91^\circ + 55.09^\circ) = 14.00^\circ
\end{aligned}$$

$$\text{Range } B^1T = \frac{B^1B^2 \sin TB^2B^1}{\sin B^1TB^2}$$

$$\begin{aligned}
\log B^2B^1 &= 3.53445\\
\log \sin 55.09^\circ &= 9.91384 - 10\\
\text{colog} \sin 14.00^\circ &= 0.61632\\
\log B^1T &= 4.06461\\
B^1T &= 11{,}604 \text{ yards}
\end{aligned}$$

$$\text{Range } B^2T = \frac{B^1B^2 \sin TB^1B^2}{\sin B^1TB^2}$$

$$\begin{aligned}
\log B^2B^1 &= 3.53445\\
\log \sin 110.91^\circ &= 9.97044\\
\text{colog} \sin 14.00^\circ &= 0.61632\\
\log B^2T &= 4.12121\\
B^2T &= 13{,}219 \text{ yards}
\end{aligned}$$

$$\text{Angle } GB^2T = \text{Azimuth } B^2G - \text{Azimuth } B^2T = 51.24^\circ$$

$$\text{Tan } \frac{1}{2}\left(B^2GT - B^2TG\right) = \frac{13219 - 3191.2}{13219 + 3191.2} \tan \frac{1}{2}\left(180^\circ - 51.24^\circ\right)$$

$$\begin{aligned}
\log 10{,}028.8 &= 4.00125\\
\log \tan 64.38^\circ &= 0.31917\\
\text{colog } 16{,}410.2 &= 5.78489 - 10
\end{aligned}$$

$$\begin{aligned}
\log \tan \frac{1}{2}\left(B^2GT - B^2TG\right) &= 0.10531\\
\frac{1}{2}\left(B^2GT - B^2TG\right) &= 51.88^\circ\\
\frac{1}{2}\left(B^2GT + B^2TG\right) &=\\
\frac{1}{2}\left(180^\circ - 51.24^\circ\right) &= 64.38^\circ
\end{aligned}$$

$$\begin{aligned}
\text{Angle } B^2GT &= 116.26^\circ\\
\text{Azimuth } GB^2 &= 276.24^\circ
\end{aligned}$$

$$\text{Azimuth } GT \quad 392.50^\circ = 32.50^\circ$$

$$\text{Range } GT = \frac{B^2T \times \sin GB^2T}{\sin B^2GT}$$

$$\begin{aligned} \log B^2T &= 4.12121 \\ \log \sin GB^2T &= 9.89195-10 \\ \text{colog} \sin B^2GT &= 0.04732 \\ \hline \log GT &= 4.06048 \\ GT &= 11{,}494 \text{ yards} \end{aligned}$$

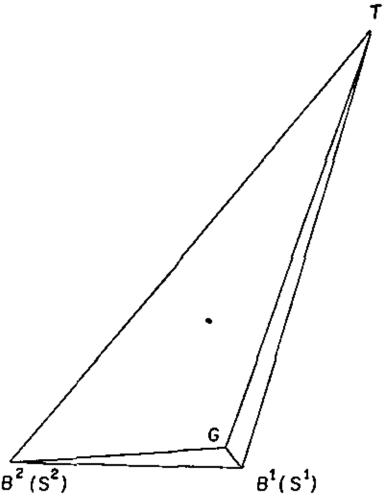


FIGURE 14.—Illustration for example, paragraph 74*b*.

■ 75. TEST OF PLOTTING BOARD.—Essentially, check points are used to detect large errors in the orientation of a plotting board, and secondly, to reduce the size of errors that may be present due to mechanical faults of the board itself. In checking the orientation of a board, check points are set on the board as stationary targets, errors tabulated, and on boards that so allow, readjustment of the orientation is accomplished to reduce the errors to a minimum in conformance with uniformity. It should be borne in mind that it is better to have a small but uniform error throughout most of the area covered by the board than to have no error at one spot and a large error at another.

CHAPTER 7

ACCURACY OF SPOTTING METHODS

SECTION I

GENERAL

■ 76. GENERAL.—Observation of fire, or spotting, is conducted for the purpose of determining the deviations of shots from the target on which to base adjustment corrections. Spotting may determine only the sense; that is, over or short, and right or left, or it may determine also the magnitude of deviations. The accuracy attained in spotting affects the promptness and adequacy of adjustment. Mistakes in sense or abnormal errors in magnitude of deviations seriously hamper adjustment.

■ 77. ACCURACY OF OBSERVATION.—The accuracy with which observed deviations are determined will depend to a great extent upon the ability of the observer. Accurate spotting is a difficult operation requiring a great deal of practice. The impact must be observed the instant it takes place and a reading made based on the position of the target at that instant. The least reading of the splash scale in observing instruments is 0.02° (some instruments have not been modified, and have graduations of 0.05°) or 5 mils, depending on the type of instrument used. An error of 0.01° in estimating the division between graduations is likely, while a further error of approximately 0.01° to 0.02° may occur in stopping the instrument at the instant of impact. It appears probable, therefore, that the accuracy with which an observed deviation is determined should be within 0.02° to 0.03°, or within a mil when using an instrument graduated in mils. Such accuracy, however, is attainable only by well-trained observers; therefore, it is impossible to put too much emphasis on training.

SECTION II

LATERAL SPOTTING

■ 78. AXIAL.—*a.* Axial spotting (angle gun–target–observing station less than 5°) of lateral deviations is used by all armament which is located where a sufficient height of instrument can be obtained to see the splashes and the target. To spot lateral deviations from an axial station, an instrument with a splash scale is necessary. The observer tracks the target until the splash occurs, when the instrument is stopped and the lateral deviation read from the splash scale.

b. The accuracy of axial spotting of lateral deviations is usually more satisfactory than that of any other method. Only one operation and one individual are involved and therefore the sources of error are reduced to a minimum. The discussion of accuracy of observed deviations in paragraph 77 is applicable to this method of spotting.

■ 79. TWO-STATION.—In this system of spotting, the lateral deviations are obtained from the operation of the spotting board. The accuracy of deviations determined is affected by the errors of two observers, by those of the spotting board operators, and by mechanical inaccuracies of the board. Obviously the measure of precision that can be obtained in this method is a function of the training of the personnel and of the mechanical adjustment of the spotting board.

SECTION III

SPOTTING BOARDS

■ 80. GENERAL.—The spotting board converts angular deviations, as read by observers, to linear deviations from the target. It is necessary that this operation is done accurately. To determine the mechanical accuracy of the board it becomes necessary to observe the results of check problems set on the board.

■ 81. CALCULATION OF CHECK PROBLEMS.—In the mathematical calculation of check problems it is usually convenient to use as basic data the check points determined for checking the plotting board. Angular deviations are then assumed, the linear deviations, right or left and over or short, are computed mathematically and compared with corresponding linear deviations determined mechanically by the spotting board. The equations given below (fig. 15) derived by differential calculus may be used in the calculation.

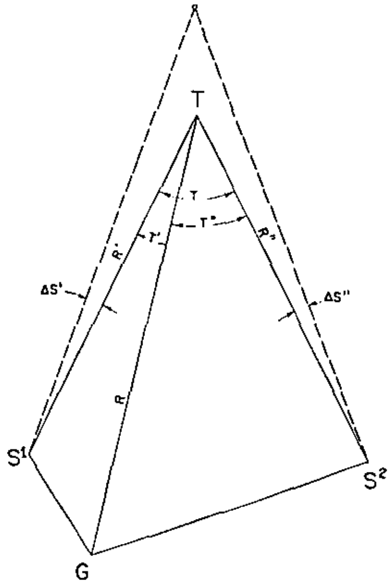


FIGURE 15.—Calculation of check points.

$$\Delta R(\text{in yards}) = \pm \frac{R' \cos T''}{\sin T} \Delta S' \pm \frac{R'' \cos T'}{\sin T} \Delta S'' \qquad (1)$$

$$\Delta L(\text{in yards}) = \pm \frac{R' \sin T''}{\sin T} \Delta S' \pm \frac{R'' \sin T'}{\sin T} \Delta S'' \qquad (2)$$

In the above equations, ΔR and ΔL are the range and lateral deviations corresponding to assumed angular deviations of $\Delta S'$ and $\Delta S''$ from spotting stations S^1 and S^2 respectively. $\Delta S'$ and $\Delta S''$ must be in radians. If they are expressed in degrees, multiply by 0.01745; if in mils, multiply by 0.0009817. In either equation, each term of the right-hand member represents the range or lateral effect caused by the angular deviation at the corresponding spotting station. The signs of the right-hand terms depend on the direction in which the angular deviation was assumed and must be determined by inspection. In figure 15, an angular deviation to the left at S^1 gives a plus sign because the effect on the range is positive. By similar reasoning, an angular deviation to the left at S^2 gives a negative sign.

■ 82. EXAMPLE.—Assume that it is desired to calculate a check problem for a spotting board of an 8-inch railway battery with the following data known (fig. 14):

	Distance Yards	Azimuth (zero north) (degrees)
S^1 to S^2	3,432.3	280.09
G to S^1	275.6	151.87
G to S^2	3,191.2	276.24
S^1 to check point No. 2	11,604	31.00
S^2 to check point No. 2	13,219	45.00
G to check point No. 2	11,494	32.00

Let the observed deviation from S^1 be right 0.15° and the deviation from S^2 be left 0.28°. What are the linear deviations from the target with respect to the directing point of the battery?

Solution:

From equation (1) above:

$$\Delta R \text{ (in yards)} = +\frac{R' \cos T''}{\sin T}\Delta S' + \frac{R'' \cos T'}{\sin T}\Delta S''$$

$R' = 11{,}604$ yards

$T'' = 45.00° - 32.50°$
$= 12.50°$

$\Delta S' = \text{right } 0.15°$

$T = 45.00° - 31.00°$
$= 14.00°$

$R'' = 13{,}219$ yards

$T' = 32.50° - 31.00°$
$= 1.50°$

$\Delta S'' = \text{left } 0.28°$

$\log R' = 4.06461$	$\log R'' = 4.12121$
$\log \cos T'' = 9.98958 - 10$	$\log \cos T^1 = 9.99985 - 10$
$\log \Delta S' = 9.17609 - 10$	$\log \Delta S'' = 9.44716 - 10$
$\log 0.01745 = 8.24180 - 10$	$\log 0.01745 = 8.24180 - 10$
$\text{colog} \sin T = \underline{0.61632}$	$\text{colog} \sin T = \underline{0.61632}$
$\log$ 1st term $= 2.08840$	$\log$ 2d term $= 2.42634$
1st term $= 123$ yards	2d term $= 267$ yards

$\Delta R = 123 + 267$ yards $= 390$ yards over

From equation (2) *above:*

$$\Delta L = -(\text{right})\frac{R' \sin T''}{\sin T}\Delta S' + (\text{left})\frac{R'' \sin T'}{\sin T}\Delta S''$$

$\log R' = 4.06461$	$\log R'' = 4.12121$
$\log \sin T'' = 9.33534 - 10$	$\log \sin T' = 8.41792 - 10$
$\log \Delta S' = 9.17609 - 10$	$\log \Delta S'' = 9.44716 - 10$
$\log 0.01745 = 8.24180 - 10$	$\log 0.01745 = 8.24180 - 10$
$\text{colog} \sin T = 0.61632$	$\text{colog} \sin T = 0.61632$
log 1st term = 1.43416	log 2nd term = 0.84441
1st term = −(right) 27 yards	2d term = +(left) 7 yards

$\Delta L = -(\text{right})$ 27 yards $+(\text{left})$ 7 yards or right 20 yards

SECTION IV

RANGE SPOTTING

■ 83. GENERAL.—There are two normal methods of range spotting; axial spotting for sense only, and two-station spotting for both sense and magnitude. Other methods of spotting range deviations may be used occasionally but only in special cases where, because of a state of emergency or unusual topography, such methods are applicable.

■ 84. AXIAL SPOTTING FOR SENSE ONLY.—Because of its simplicity and speed, this method is used for rapid-fire guns. The observer, from an axial observing station, tracks the target with an instrument and locates the splash as over or short of the target. The range sensings are accurate only when the splash is in line with some portion of the target. When this is the case, it can be definitely determined whether the splash is over or short. Attempting to sense shots for range when the splash is not in line with the target (note below) is bad practice and should not be permitted unless the height of site of the station is such that the relative positions of the splash and the target are prefectly clear. Since lateral deviations are usually small, all but a few splashes may be expected to be in line with a portion of the target and sensings may be determined without difficulty.

NOTE.—Since splashes will very seldom be in line with a pyramidal target, due to the small size of the latter, instructions for target practice authorize the use of two-station (or flank) spotting to obtain range sensings.

■ 85. Two-Station Magnitude Spotting.—This system of range spotting employs a spotting board and two observing instruments at the ends of a base line. Observers track the target with their instruments until the splash occurs, then stop tracking and read the angular deviation from the target. On the spotting board, the observations are plotted and the location of the splash with respect to the target determined. The magnitude of the deviation as well as the sense is read from the spotting board. The accuracy of this method of spotting is dependent upon the skill and training of the personnel and the mechanical accuracy of the spotting board. With proper attention to these factors, the results using this method of spotting are well within the necessary limits of accuracy.

■ 86. Spotting by Depression Position-Finder.—Efforts are frequently made to determine the magnitudes of range deviations by the use of a D. P. F. in an axial station, taking the differences between ranges measured to the target and those measured to the splashes. Except under exceptional circumstances the method is considered unsatisfactory because of the difficulty in water lining a splash. Before a battery commander decides to use such a method of spotting, he should be sure that the height of site of the instrument will give sufficient accuracy at the ranges to be used. In making an analysis of the situation, the equation in paragraph 67*b* may be used, converting the mean angular error of the observer into the mean error in yards at the expected range. In this connection, the mean angular error of an observer in all probability will be two or three times greater when observing on splashes than when observing on targets, and this fact should be given consideration when employing the abovementioned equation. Since the task of observing alternately on the target and a splash would cause large irregularities in the plotted course, the same observer never should be required both to read range deviations and to furnish data for the plotting board.

■ 87. Special Methods.—There are other methods of spotting range deviations which may be applicable in situations where there is particularly favorable topography. For example, if an axial station has a great height of site, the observer may sense overs and shorts by keeping the horizontal cross wire of the instrument on the water line of the target and by noting whether the splash is above or below that wire. Obviously, to sense small range deviations, a great height of site is required for the use of such a method.

CHAPTER 8

CANT AND SIGHT DISPLACEMENT

SECTION I

CANT

■ 88. GENERAL.—*a.* Cant is the inclination from the horizontal of the trunnions about which a cannon is rotated for pointing in elevation. If these trunnions are not level, the axis of the bore will not remain in a vertical plane as the cannon is elevated. This will change the azimuth at which the cannon is pointing. At the same time, it will cause an error in the quadrant elevation if the angle through which the cannon has been elevated was not measured in a vertical plane. This latter error is, however, negligible for angles of inclination less than 4°.

b. The magnitude of the effect of cant depends on the inclination of the axis of the trunnions and the quadrant angle of elevation. If the inclination is denoted by I, the quadrant elevation by ϕ, the angular error of pointing in direction by d_I, then

$$d_I = I \tan \phi$$

■ 89. CORRECTIVE MEASURES.—Fixed seacoast guns are mounted on permanent platforms so that the carriages, and consequently the axes of the trunnions, are level or nearly so. The sight mount for such a gun is rigidly attached to the carriage. Upon assembly to the carriage, it is adjusted so that the sight which it supports will read horizontal angles and, if required, vertical angles. If a deflection is set on the sight and the gun traversed until the line of sight includes the aiming point, the axis of the trunnions is given a definite direction. Since the axis of the bore is perpendicular to the axis of the trunnions, movement of the cannon in elevation will cause the axis of the bore to sweep out a plane perpendicular to the axis of the trunnions and, since the axis of the trunnions is horizontal, this plane will be vertical. The axis of the bore will, therefore, have a fixed direction with respect to the line of sight regardless of the angle of elevation. Due to the fact that mobile guns are seldom mounted on level platforms, the sight mounts for this matériel include means for establishing a horizontal line perpendicular to the axis of the bore and for giving that line a definite direction. These instruments are called "compensating sight mounts."

SECTION II

COMPENSATING SIGHT MOUNTS

■ 90. CONSTRUCTION.—There are several types of compensating sight mounts issued for use with mobile guns. They differ somewhat in the details of construction but are all similar in one respect, that is, the use of a "cross." For purposes of identification, the arms of this cross may be called the "long axis" and the "short axis." The sight mount is attached to the gun or carriage so that the long axis is maintained parallel to the axis of the bore. (See fig. 16.) This is accomplished either by attachment through a bracket to a gun trunnion or by means of a linkage between the long axis and the trunnion. The short axis is made perpendicular to the long axis and a sight is mounted on a shank perpendicular to the short axis. Two levels are attached to the sight mount, one parallel to the short axis and the other perpendicular to it and to the sight shank. By means of these levels, the short axis may be maintained horizontal and the sight shank vertical. Thus we have a means for establishing a horizontal line perpendicular to the long axis regardless of the angle of elevation of that axis, and for giving this line a definite direction.

As the gun is elevated, the long axis of the cross will be elevated and pulled away from its original azimuth. (See fig. 17.) The short axis will be rotated through a horizontal angle equal to that swept through by the long axis of the cross and the axis of the bore. Since the sight shank is mounted on the short axis and in such a manner that it may be maintained vertical, the line of sight will be traversed through the same horizontal angle as the short axis. Then by traversing the gun until the line of sight is back on the aiming point, the gun is brought back to its original azimuth.

■ 91. SOURCES OF ERROR.—It is obvious that in a properly adjusted sight mount of the compensating type the leveling mechanisms perform two important functions. When the bubbles are centered, they establish a horizontal plane parallel

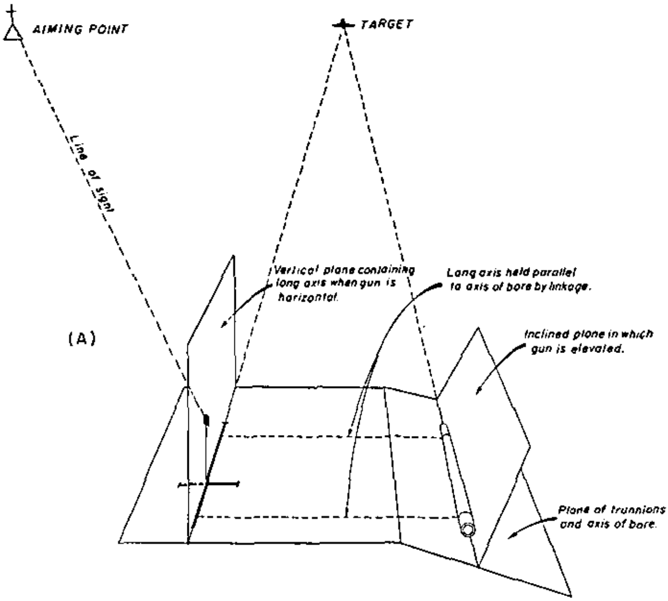


FIGURE 16.—Principle of compensating sight mount, gun in horizontal position.

to the axes of the levels. This results in making the short axis horizontal and in making the sight shank vertical. The first function compensates for cant and is by far the more important. The second function merely assures the measurement of horizontal angles on which small errors in the verticality of the sight will have little effect unless the aiming point is at a considerable angle above or below the sight.

Some of the maladjustments of the sight mounting which may cause errors are—

a. Levels out of adjustment (the most likely source of error).

b. Sight shank loose or bent.

c. Distorted or improperly assembled sight bracket.

d. Lack of perpendicularity of members of the cross.

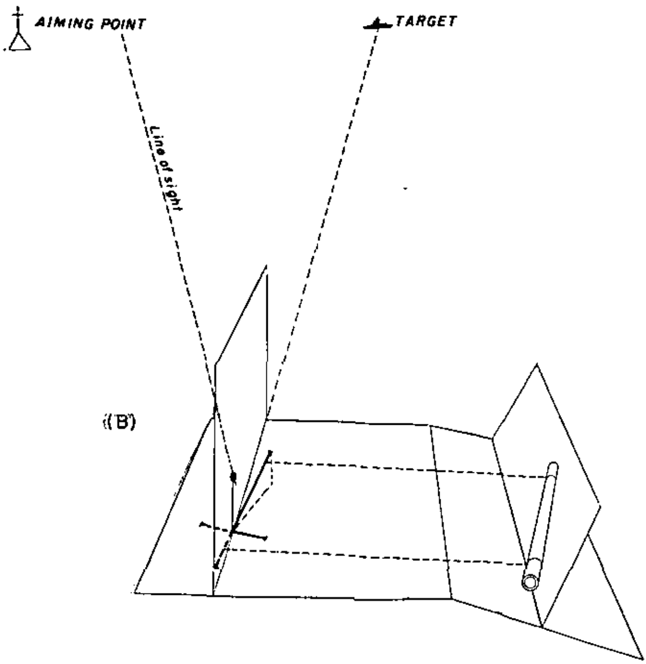


FIGURE 17.—Principle of compensating sight mount, gun in elevated position.

■ 92. ANGULAR ERROR.—*a*. To understand the angular error caused by faulty adjustment of the cross level bubble, it is only necessary for one to visualize the gun and cross of the sight mount interchanged in position in figures 16 and 17, so that the short axis of the cross lies in the inclined plane and the trunnions of the gun lie in the horizontal plane. As the gun is elevated, it will follow the vertical plane and cause the long axis of the cross also to follow a vertical plane. Bearing in mind that the short axis remains in the inclined plane, it may be seen that as the long axis is elevated the short axis must rotate through an angle in the inclined plane, thus changing its azimuth and that of the line of sight. If the gun is now traversed to bring the line of sight back on the aiming point, it will introduce an error in azimuth. Assuming, with a compensating sight mount, that the axis of the bore is always parallel to the long axis, then the error in pointing in direction due to failure to level the short axis is the same as for a corresponding inclination of the axis of the trunnions, that is

$$d_1 = I \tan \phi$$

b. If the axis of the bore is not parallel to the long axis, the entire principle of the compensating sight mount is violated and the center line of the bore at any angle of elevation will be parallel to an element of a cone whose axis coincides with the short axis of the cross. The magnitude of the resulting error in pointing (note below) depends on the quadrant elevation and the amount of divergence between the axis of the bore and the long axis of the cross. If the lack of parallelism is appreciable it will become apparent when trying to readjust the cross level bubble. (See pertinent Technical Manuals.) If this condition is found, it will be necessary to refer the matter to the ordnance repair section for repair.

NOTE.—If d_L denotes the angular error of pointing due to lack of parallelism, then

$$d_L = L \text{ exsec } \phi$$

in which L is the angular value of the lack of parallelism in degrees and ϕ is the quadrant elevation. If tables of external secants are not at hand, exsec ϕ may be expressed as $\frac{1}{\cos \phi} - 1$.

■ 93. METHOD OF CHECKING ADJUSTMENT OF CROSS LEVEL BUBBLE.—The Ordnance Department has a very accurate method of checking the cross level bubble of the sight mount by actually leveling the trunnions of the gun, thus making the axis of the bore follow a vertical plane and remain at the same azimuth while being elevated. However, satisfactory results can be obtained in the field by causing a vertical cross hair on the muzzle of the gun to follow a plumb line as the gun is elevated. The operation of checking the cross level bubble using this system is as follows:

a. Place a cross wire on the vertical center line of the muzzle.

b. Establish a plumb line in front of the muzzle.

c. With the bore approximately horizontal, boresight the gun on the plumb line.

d. Center both longitudinal and cross levels.

e. Move the sight head and place the line of sight on a distant aiming point.

f. Elevate the gun as high as possible to magnify any error that may be present and again boresight on plumb line, traversing the gun if necessary.

g. Check level of sight, releveling if the bubbles are not centered.

h. Check position of the line of sight. If it is on the aiming point, this is an indication that the level bubble is properly adjusted. If it is not on the aiming point, the cross level bubble is out of adjustment and must be readjusted. (In exceptional cases, it may be that the sight mount in some way has been damaged.) If the left end of the short axis is low, the line of sight is to the right of the aiming point; and if the right end of the short axis is low, the line of sight is to the left of the aiming point. It follows, therefore, that in normal operation of the sight, if the gun is fired with the left end of the short axis low, the error in pointing is to the *left;* and if the right end is low the error in pointing is to the *right.*

■ 94. EXAMPLES.—*a.* The base ring of a 6-inch gun on a disappearing carriage has settled so that the trunnions are canted 3 minutes when firing at an azimuth of 270°. The right trunnion is higher than the left. Is a deflection correction for cant necessary?

Solution:

$d_I = I \tan \phi$, where the maximum value of ϕ is 15° and

$I = 3 \text{ minutes} = 0.05°$

$d_I = 0.05° \times \tan 15° = 0.05° \times 0.27 = 0.01°$

and the error is negligible.

b. The cross level on the sight mount of an 8-inch gun on railway carriage, M1918, is out of adjustment so that when the bubble is centered, the short axis of the sight mounting is

30 minutes out of level, the right end being low. In other respects, the sight mounting is correctly adjusted. What will be the lateral effect on a shot fired at an elevation of 25°?

Solution:

$$d_I = I \tan \phi$$

Upon substitution of the given values,

$$d_I = 0.50° \tan 25° = 0.50 \times 0.466 = 0.23°$$

Since the right end of the short axis is low, the effect will be 0.23° to the right.

SECTION III

SIGHT DISPLACEMENT

■ 95. GENERAL.—*a*. Sight displacement is the horizontal distance between the vertical axis of the sight and the vertical axis about which the gun is traversed, that is, the pintle center.

b. In the case of a mobile gun equipped with an oriented and properly adjusted panoramic sight, the sight reading, when the line of sight is directed at an aiming point at an infinite distance, will always be the azimuth of the vertical plane containing the axis of the bore. Due to the displacement of the vertical axis of the sight from the vertical axis about which the gun is traversed, the use of an aiming point at a finite distance will result in parallax. The amount of this parallax will depend upon the distance to the aiming point, distance between the two vertical axes, and direction of fire. For a given aiming point and a given gun, the parallax will be zero only when the aiming point and the two vertical axes lie in the same vertical plane; it will be a maximum when the gun is traversed 90° either to the right or left of the position which has zero parallax.

■ 96. ERROR IN POINTING DUE TO SIGHT DISPLACEMENT.—*a*. No azimuth error is present when the gun is laid at a known azimuth and the sight is set to read that azimuth when the line of sight is directed at the aiming point. When these conditions exist, the sight is said to be oriented.

b. Traversing the gun to any other azimuth develops an error in azimuth due to the displacement of the sight. For example, in figure 18, G represents the pintle center of a gun, *S* the sight, and *P* the aiming point. If the sight were directed at the aiming point and set to read the azimuth of the axis of the bore while in this position, and then the gun traversed to the position shown by the dotted lines without turning the head of the sight, the new line of sight will be *S'D*. If the sight head is turned so that the new azimuth of the bore is indicated, the line of sight will be *S'E*. Since the gun and sight were both turned through the same angle, all angles marked α in the figure are equal and *S'E* is parallel to *GP*. The parallax error at the new azimuth is then the angle *ES'P* and in pointing the sight at the aiming point without turning the sight head, the gun will be traversed too far to the left.

■ 97. CALCULATION OF ERRORS.—In calculating the amount of error due to sight displacement, the effective sight displace-

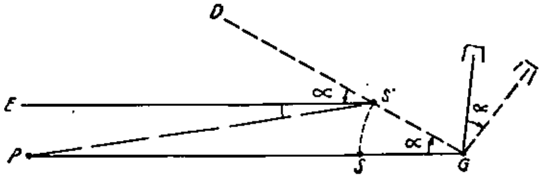


FIGURE 18.—Sight displacement.

ment must be determined for each position of the gun. This is the perpendicular distance from the vertical axis of the sight to the line from the pintle center of the gun to the aiming point. Obviously, when the sight remains on the same side of the line gun–aiming point as it was in the orienting position, the error in laying the gun is equal to the parallax due to the difference between the effective sight displacement at the azimuth of the target and that at the orienting azimuth. If the gun is traversed until the sight is on the opposite side of the line gun–aiming point from the orienting position, the error in laying is the parallax due to the sum of the respective effective sight displacements. In calculating the parallax in degrees due to displacement of the vertical axis of the sight from the pintle center of the carriage, the following approximate rules may be used: Determine the effective displacement of the sight from its orienting position; then divide the effective displacement of the sight by the distance to the aiming point and multiply by 57. The result will be in degrees. The same unit must be used in measuring both distances. If the result is desired in mils, multiply by 1,000 instead of 57.

■ 98. SELECTION OF AIMING POINT.—*a*. If the aiming point can be selected so that the maximum sight displacement is less than 1/2,000 of the distance to the aiming point, the maximum error due to displacement of the sight will be less than 0.03° and may be neglected. For other situations, consideration should be given to this source of error to insure satisfactory accuracy in important sectors of the field of fire. In some situations, due to the limited extent of the important portion of the field of fire, the errors due to sight displacement may be found to be negligible or may be made so by introducing a parallax correction into the orienting data. In other situations, more than one aiming point may be established and the sight reoriented when a shift is made from one part of the field of fire to another. In some cases, the only satisfactory solution will be the use of the aiming rule.

b. In general, if the aiming point is on a line at right angles to the line from the pintle center to the sight in orienting position, the error due to sight displacement will not be appreciable until the gun is traversed beyond 45° on either side of the orienting azimuth. The error will then increase rapidly, reaching the angle corresponding to the full displacement of the sight at 90° on either side and increasing to twice this error at 180° from the orienting azimuth. On the other hand, if the aiming point is on the prolongation of the line from the pintle center to the sight, in orienting position, the error will increase rapidly for azimuths up to 45° on either side of the orienting azimuth and will then continue to increase, but much less rapidly, and will never exceed the parallax corresponding to the full displacement of the sight which will be reached at 90° from the orienting azimuth. The problem may be better understood from the examples in paragraph 99.

■ 99. EXAMPLES.—*a*. (1) The vertical axis of the sight on a 12-inch railway mortar carriage, M1918, is 23.25 inches in front and 48.5 inches to the left of the pintle center. If the azimuth to the center of the field of fire is 100° and in this position the aiming point is on a line to the rear of the mortar and at right angles to the line from the pintle center to the sight, what is the azimuth of the line *sight—aiming point?*

Solution (fig. 19Ⓐ):

$$\text{tan angle } \textit{muzzle-pintle center-sight} = \frac{48.5}{23.25} = \tan 64°\ 23.3'$$

Angle *muzzle-pintle center-sight*=64°23.3'=64.39°.
Azimuth *pintle center-sight*=100°−64.39°=35.61°.
Azimuth *sight-pintle center*=35.61+180°=215.61°.
Azimuth *sight-aiming point*=215.61°+90°=305.61°.

(2) What is the distance *pintle center-sight?*

Solution (fig. 19Ⓐ):

$$\frac{48.5''}{\sin 64°23.3'} = 53.78 \text{ inches or } 1.49 \text{ yards}$$

(3) If the mortar is pointed at azimuth 155°, what parallax error is introduced? (Use 1.5 yards as the distance pintle center-sight, and 1,000 yards as the distance to the aiming point.)

Solution (fig. 19 Ⓑ): In this case, the mortar has been traversed 55° to the right and the sight is on the same side of the line gun-aiming point as it was in the orienting position; the parallax error is that caused by *S'E*, the difference between the effective sight displacement at the orienting position and that at the new position. That is—

Effective sight displacement at orienting position=1.5 yards.

Effective sight displacement at azimuth 155°=1.5×cos 55°

$S'E$=1.5−1.5 cos 55°=1.5 (1−cos 55°).

$$\text{Error} = \frac{1.5(1-\cos 55°)}{1{,}000} \times 57 = 0.04°$$

The mortar will be pointed 0.04° too far to the right.

(4) If the mortar is traversed to azimuth 210°, what parallax error is introduced?

Solution (fig. 19 Ⓒ): In this case, the mortar has been traversed 110° to the right and the sight is on the opposite side of the line mortar-aiming point from the orienting position, and the parallax error is equal to the angle subtended by $S'E$, the sum of the respective sight displacements.

$$\frac{1.5+1.5\times\sin 20^\circ}{1{,}000}\times 57=0.11^\circ$$

The mortar will be pointed 0.11° too far to the right.

(5) At any particular azimuth, how may corrections be made for the parallax error which exists at that azimuth?

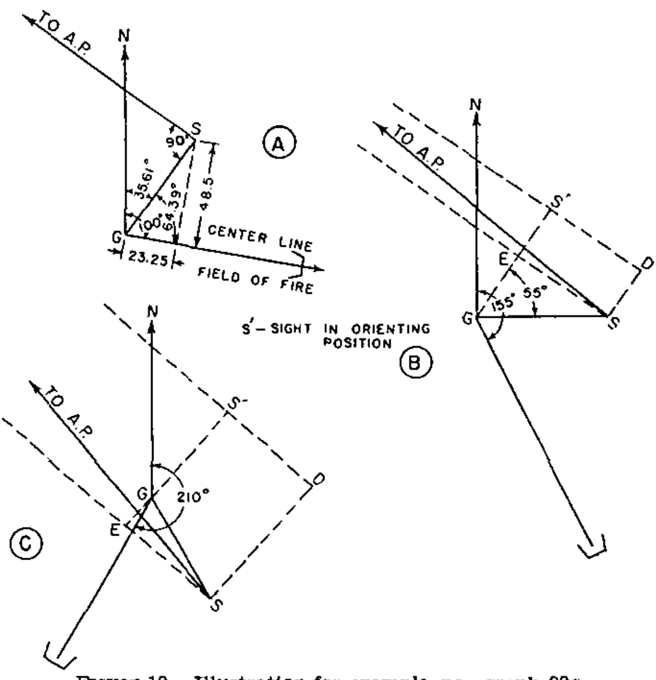


FIGURE 19.—Illustration for example, paragraph 99*a*.

Solution: Set the mortar at the desired azimuth and make the sight indicate that azimuth when the sight is accurately on the aiming point.

b. (1) The vertical axis of the sight on an 8-inch railway gun carriage, M1918, is 5.75 inches forward, and 33.625 inches left, of the pintle center of the carriage. What is the difference between the azimuth of the axis of the bore and of the line from the pintle center to the sight?

Solution (fig. 20 Ⓐ):

$$\tan \textit{muzzle-pintle center-sight} = \frac{33.625}{5.75}$$
$$= \tan 80° \ 17.8'$$
$$= \tan 80.30°$$

(2) What is the distance from the pintle center to the sight?

Solution (fig. 20 Ⓐ):

$$\frac{33.625}{\sin 80° \ 17.8'} = 34.113 \text{ inches or } 0.95 \text{ yards}$$

(3) The gun is boresighted near the center of the field of fire and the sight is oriented on an aiming point 400 yards to the left of the gun and in prolongation of the line from the pintle center to the sight. The gun is then pointed by the sight 30° to the left. What error due to displacement of the sight is introduced?

Solution (fig. 20 Ⓑ): In this position, the effective sight displacement is equal to the distance *pintle center-sight* times the sine of 30°, and the error thus introduced is equal to—

$$\frac{0.95 \sin 30°}{400} \times 57 = 0.07°$$

The gun will be pointed 0.07° too far to the right.

(4) The gun is pointed by the sight 90° to the right of the orienting position. What is the error?

Solution (fig. 20 Ⓒ): In this position, the displacement of the sight has its maximum parallax effect, since the effective displacement ($S'D$) is equal to the total displacement (GS); the error is equal to—

$$\frac{0.95}{400} \times 57 = 0.14°$$

The gun will be pointed 0.14° too far to the left.

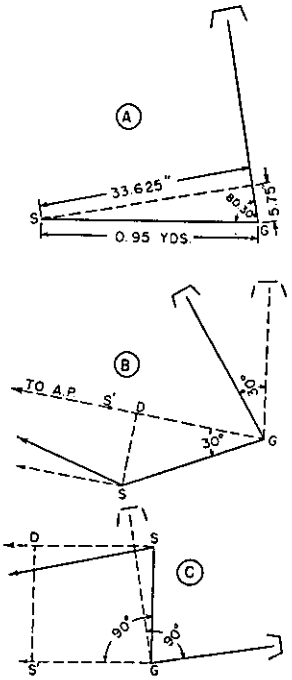


FIGURE 20.—Illustration for example, paragraph 99*b*.

CHAPTER 9

DISPERSION AND ERRORS

SECTION I

GENERAL

■ 100. GENERAL.—Individual shots in artillery fire rarely fall on the spot calculated, even though great skill and care are used in firing the guns. However, a knowledge of the factors which cause this variation from the intended point of impact serves as a basis for proper analyses and will materially aid an artilleryman in increasing the effectiveness of his fire. This chapter describes briefly the characteristics of artillery fire and gives the causes and remedies for certain conditions which are detrimental to the accuracy and effectiveness of artillery fire.

SECTION II

DISPERSION

■ 101. GENERAL.—*a*. If several shots are fired from a gun laid each time at the same azimuth and elevation, these shots will not all fall at the same spot. They will be scattered, both in range and in direction, by unavoidable changes in pointing, muzzle velocity, wind, and all other things that determine the shape of the trajectory. This scattering is called "dispersion."

b. The area over which shots are scattered by dispersion is called the "dispersion zone." The distribution of shots within the dispersion zone is a matter of importance in drawing up rules for adjusting fire, estimating fire effect, and calculating ammunition requirements. The subparagraphs below show what is generally accepted as true regarding this distribution, giving the combined results of experience and methematical theory. It is neither necessary nor desirable that the theoretical bases be stated fully; the results alone are sufficient for the purposes of this manual.

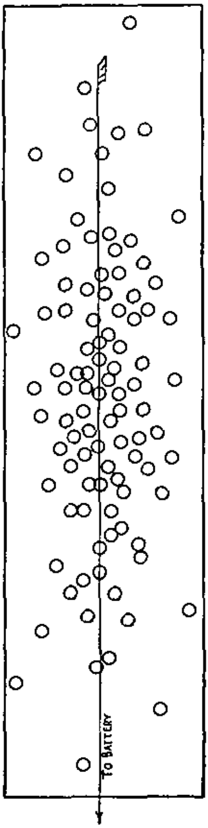


FIGURE 21.—The 100 percent rectangle.

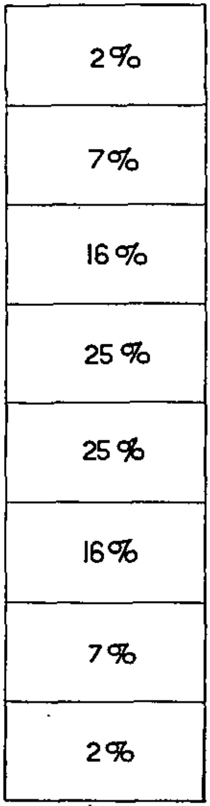


FIGURE 22.—Dispersion ladder for range.

c. If a great many shots are fired with the same pointing, it is probable that a plot of the points of impact will show the same general characteristics as the group plotted in figure

21. The concentration is densest near the center of the group and becomes gradually less toward the outer edges. There are as many shots short of the center of the group as there

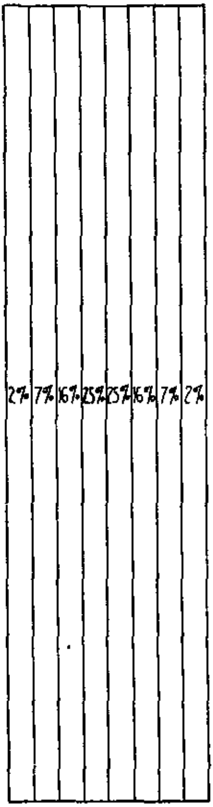


FIGURE 23.—Dispersion ladder for direction.

FIGURE 24.—Dispersion diagram.

are beyond it, and as many to the right as to the left. Range dispersion is almost always greater than lateral dispersion, so that the longer axis of the group is along the gun-target line.

d. A rectangle constructed as in figure 21 so as to include all or practically all of the shots is called the "100 percent rectangle." If the 100 percent rectangle is divided into eight equal parts by lines drawn perpendicular to the line of fire as in figure 22, the percentage of shots to be expected in each part is that indicated in the figure. This figure is called a "dispersion ladder." The dispersion ladder may be expanded or contracted to make it fit various conditions of dispersion, but the lengths of the divisions always remain equal, one to another, and the percentages do not change. In figure 22, the dispersion ladder has been constructed to show range dispersion and may be called the dispersion ladder for range. Figure 23 shows the construction of a dispersion ladder for direction, the dividing lines being drawn parallel to the line of fire.

e. The two strips lying nearest the center of the dispersion ladder are expected to contain one-half of all shots fired. Together they make up the "50 percent zone." There is a 50 percent zone for range and a 50 percent zone for direction. In some cases, it is necessary to specify which of the two is intended.

f. The distance equal to one-half the length (or width) of the 50 percent zone is called the "probable error." Until the meaning of the word "error" has been defined, the term "probable error" should be interpreted as simply a unit of distance to be used in specifying the size of the 100 percent rectangle. The range probable error is one-eighth of the length, and the lateral probable error is one-eighth of the width of that rectangle.

g. When the dispersion ladder for range is superimposed upon that for direction, the result is the assemblage of small rectangles, shown in figure 24, called the "dispersion diagram." The percentage of shots to be expected in any particular small rectangle is derived from the percentages expected in the two strips, range and lateral, whose intersection forms the rectangle. For example, a rectangle lying in one of the 7 percent strips of the dispersion ladder for range and also in one of the 16 percent strips of the dispersion ladder for direction would be expected to include 16 percent of 7 percent of the shots, or about 1 percent ($0.16\times0.07=0.0112$).

■ 102. DEVIATIONS.—*a*. The *deviation* of a shot is the distance by which it misses the target or other point it was intended to hit. The *absolute deviation* is this distance as measured along a straight line drawn from the point of impact to the target, as in figure 25. The absolute deviation is seldom used as such, but is generally broken up into its two components, *range deviation* and *lateral deviation*. The range deviation is measured along a line parallel to the gun–target line and the lateral at right angles to it.

b. The *center of impact*, or mean point of impact, of a series of shots is a point whose position is fixed by the positions of the several points of impact. The range deviation of the center of impact is the algebraic mean of the range deviations of the separate lateral deviations.

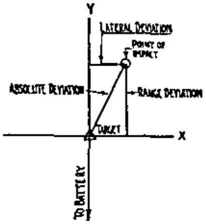


FIGURE 25.—Deviations.

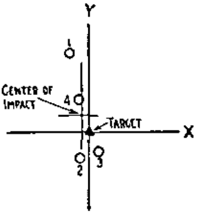


FIGURE 26.—Center of impact.

c. It is convenient to refer deviations and the center of impact to a set of rectangular axes in the horizontal plane containing the target. The intersection of the axes is placed at the target and the Y-axis is placed along the gun-target line (the X-axis being perpendicular thereto). The range deviation of a point of impact is then the Y-coordinate of that point, and its lateral deviation is the X-coordinate. The Y-coordinate of the center of impact is the *algebraic mean* of the Y-coordinates of the points of impact, and the X-coordinate of the center of impact is the algebraic mean of the several X-coordinates.

d. For example, four shots are fired with the same pointing and fall as indicated in the table below. These points of impact are plotted in figure 26.

Shot No.	Range deviation (yards)	Lateral deviation (yards)
1	Over 180	Left 42.
2	Short 60	Left 18.
3	Short 45	Right 24.
4	Over 73	Left 22.

The Y-coordinate of the center of impact is $(180-60-45+73)/4=148/4=+37$ yards. The X-coordinate of the center of impact is $(-42-18+24-22)/4=-58/4=-14.5$ yards. It is therefore 37 yards beyond the target and 14.5 yards to the left.

e. Up to this point, the only kind of dispersion considered has been that of a gun fired each time with the same pointing. In actual artillery problems, it is necessary to deal with the dispersion of guns firing at moving targets when personnel errors and adjustment corrections are being made. The difficulty introduced by the target's moving is overcome by working only with deviations, ignoring the fact that range and direction are changing. The effects of adjustment corrections and known personnel errors may be eliminated by calculating for each shot the deviation that would have occurred if these disturbing influences had not been present. If these steps are not taken, the term "center of impact" must be given an entirely different interpretation from that intended in the preceding subparagraphs.

f. The center of impact must be distinguished from the *center of dispersion.* The center of impact is the mean point of impact of shots that have already fallen; the center of dispersion is entirely a theoretical point, the center of the group that would have been formed if an infinitely large number of shots had been fired. Obviously, an artilleryman can never locate the center of dispersion and, for all practical purposes in any operation or discussion, he must consider the center of impact only, treating it as though it were actually the center of dispersion.

g. When a shot is erratic or wild it is disregarded in the calculation of the center of impact, probable error, adjustment correction, or any similar data. The rules for determining whether or not a shot is wild are based on the general proposition that if it falls outside the 100 percent rectangle, or more than four developed probable errors from the center of dispersion, it should be disregarded. The detailed rules for making the determination are changed from time to time and are different for different purposes.

SECTION III

DEFINITIONS OF ERRORS

■ 103. ERROR.—An error is the difference between the observed or calculated value and the true value of a quantity. In the language used by artillerymen, the word means a departure from normal or standard. Different kinds of errors have been given distinctive names and a knowledge of the special meanings conveyed by these names is necessary.

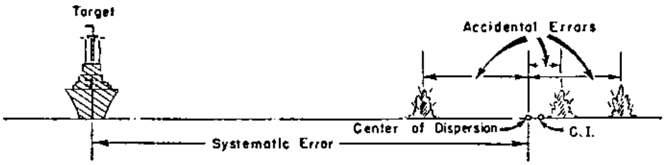
■ 104. PERSONNEL ERRORS.—These are minor divergences from absolute precision made by the personnel in the operation of fire-control instruments. Small personnel errors are unavoidable. Large personnel errors, generally called "mistakes," can be avoided by proper care.

■ 105. SYSTEMATIC ERROR.—A systematic error is an error affecting all readings in a series alike. In artillery firing, the divergence of the center of impact of a large number of shots from the target is caused by a systematic error. The term is used in this way by analogy with its use in the study of precision measurements, where it means the divergence of the average of several readings from the true value of the quantity measured.

■ 106. ARMAMENT ERROR.—The armament error is a special term meaning the divergence, stripped of all personnel errors and adjustment corrections, of a shot from the center of impact of a series of shots similarly stripped. In the analysis of a firing, the mean armament error (arithmetical mean; that is, disregarding all algebraic signs) is multiplied by 0.845 to obtain what is known as the "developed armament probable error (DAPE)."

■ 107. ACCIDENTAL ERRORS.—These are unpredictable variations from normal that cannot be entirely eliminated and are irregular in their effect on consecutive trials. In the accidental errors of a series of trials, it is found that the errors are more likely to be small than large, just as likely to be plus as minus, and by experiment a maximum limit can be determined which no error should exceed. In gunfire, the divergence of the individual impacts from the center of dispersion (practically, from the center of impact) is the result of accidental errors.

■ 108. PROBABLE ERROR.—The probable error is that accidental error which is as likely as not to be exceeded on any one trial. In gunfire, the probable error is that distance from the center of impact of a large number of shots to point of impact which is as likely as not to be exceeded on any one shot, and by reference to figure 22, it will be seen to be ⅛ of the 100 percent zone. In the ideal dispersion group, one half of



For practical purposes the center of impact is considered the center of dispersion.

FIGURE 27.—Systematic and accidental errors.

the shots lie more than one probable error and the other half lie less than one probable error from the center of impact. Since those lying more than one probable error from the center of impact are much more scattered than those lying closer, the average error, or *mean error*, is greater than the probable error. The ratio between the two is fixed and the probable error is equal to the mean error multiplied by 0.845.

SECTION IV

CAUSES OF ERROR

■ 109. GENERAL.—A great number of factors affect artillery fire and the elimination of all errors is impossible. In the calculation of firing data, errors are introduced by the inaccuracies of instruments, and by mistakes, ignorance, and carelessness of personnel. These errors may be classified as both accidental and systematic errors and are reduced to a minimum by training and care. Other sources of error fall into one of three general groups; conditions in the carriage, conditions in the bore, and conditions during flight.

■ 110. CONDITIONS IN THE CARRIAGE.—Variations in the accuracy of laying and nonuniformity of reaction to firing stresses cause accidental errors. Physical limitation of precision in setting scales, play in the gearing, and in mobile guns, differences in the footing of spades and outriggers, are factors which cannot be entirely eliminated. To reduce the variable effect of these conditions from round to round, settings should be made from the same direction each time; matériel should be maintained in excellent condition; and, in the case of mobile guns, spades and outriggers should be carefully and solidly emplaced.

■ 111. CONDITIONS IN THE BORE.—Variations in conditions inside the bore while firing contribute accidental and systematic errors of great magnitude. Variations in temperature, composition of powder, ignition, weight of projectile, erosion, and density of loading have a direct effect on muzzle velocity and consequently on the range.

a. As discussed in chapter 3, the temperature of the powder charge as the gun is fired determines to a great extent the muzzle velocity to be expected. If it is different from the temperature assumed in the calculation of firing data, a systematic error will result. Variation in the temperature of individual powder charges may be caused by differences in the length of time the powder charges remain out of the powder magazine, by differences in the time the charges remain in the bore before firing, and by differences in temperature of the bore. Such variations cause small changes in the developed muzzle velocity and consequent accidental errors, the size of which may be reduced by uniform handling of powder charges.

b. The composition of the powder is subject to change during storage due to the change in moisture content and the loss of volatiles. This will cause a change in muzzle velocity and a resultant systematic error. If the storage conditions are particularly unfavorable, the powder may deteriorate sufficiently to become dangerously unstable. No method has been devised to measure the effects of changes in the composition of the powder; the only effective action that can be taken by a battery commander is to make conditions as favorable as possible. Powder containers should be handled carefully to prevent breakage of the airtight seal. Charges should be segregated into lots and each lot stored under the same conditions. When preparing for a firing, the charges should be kept under shelter and not opened until needed.

c. Nonuniform ignition of the propelling charge causes variations in muzzle velocity and consequent accidental errors. Uniformity of burning of powder charges has been the subject of considerable study by the Ordnance Department, and many advances have been made to reduce the size of errors caused by this factor. Igniting charges are scientifically incorporated in the powder charge to insure complete and uniform ignition and, in guns of 8-inch and greater caliber, the present practice is to use the stacked type of charge in which the powder grains are stacked end to end in a precise manner. These precautions have resulted in a very substantial reduction in the magnitude of accidental errors. Action that can be taken by an artillery commander to reduce further the size of accidental errors is to make sure that all powder charges of a lot are uniform in size and weight and that igniters are properly placed in the charge. In addition, proper gunnery methods require that the breechblock push the powder charge into the powder chamber during the last short distance of its forward motion in order that the igniter pad will be against the mushroom head to obtain the full effect of the hot gases from the primer as it is fired.

d. Erosion is the wearing of the interior of a gun tube due to the action of the projectile, powder grains, and powder gases. It causes a loss of muzzle velocity and results mostly in a systematic error. It is possible to calculate the approximate loss of velocity due to erosion by means of star gaging and the use of formulae and charts, but the procedure is not common practice under service conditions. Since guns are not eroded equally, the unequal loss of muzzle velocity makes it necessary to apply corrections to individual guns to make them all shoot together. This is called "calibration." (See sec. V.) A battery commander should avoid the need for calibration corrections by trying to keep the number of rounds fired by each of the guns equalized.

e. One of the factors on which the developed powder pressure and therefore the muzzle velocity depends is the *density of loading*. It is the ratio of the weight of the powder charge to the weight of the volume of distilled water at 39.2° F. that 4°C would fill the powder chamber. Obviously the two elements that would cause variations in the density of loading are non-uniformity of weight of powder charges and differences in the seating of the projectiles, that is, the positions in the bore just before firing. Either of these two elements will cause small changes in muzzle velocity and corresponding accidental errors. Hence, it becomes necessary to seat the projectiles as uniformly as possible. This is best accomplished by causing the ramming detail always to use the maximum force in ramming. Incidentally, this will prevent the projectile from slipping back against the powder when the cannon is elevated, an event which might cause dangerous pressure.

f. The weight of the projectile, as discussed in chapter 3, has an effect on the muzzle velocity and on the ballistic coefficient of the projectile. These effects are contradictory in their influence on the range obtained and their resultant magnitude changes with different ranges. Variations in the weight may cause systematic errors and accidental errors. If the weight of projectile assumed in the calculation of firing data differs from the mean weight of the projectiles fired, a systematic error is introduced. Variations in weight of individual projectiles contribute accidental errors. To reduce these errors to a minimum, projectiles should be segregated into groups by weight and firings executed with projectiles all from the same group. Correction is then made in the firing data for the variation from standard weight.

■ 112. **Conditions During Flight.**—During the time the projectile is in the air, there are many conditions that operate to cause errors. Some of these are the ballistic properties of the projectile, the angle at which it leaves the bore of the gun, and atmospheric conditions encountered by the projectile. Of these, the last is probably of most importance to the artilleryman. The meteorological data used as a basis for correction of firing data are seldom exactly correct, and the discrepancies cause both accidental and systematic errors. Rarely, if ever, will two shots from the same gun pass through identical conditions of wind in their trajectories. Therefore accidental errors take place. Again, it is quite likely that differences exist between the actual effective conditions at the time of a firing and those described by the meteorological message. This situation presents a systematic error in the firing data. Since it is impossible to determine the magnitude of these errors, the only solution is to use meteorological data that are as recent as possible.

■ 113. SEGREGATION OF ERRORS BY ANALYSIS.—Prior to firing artillery, a study should be made to determine sources of errors and steps should be taken to reduce the errors to a minimum. Those not eliminated will appear during actual firing and, if proper records are kept, an analysis of the results can be made. The purpose of the analysis is to reduce the size of errors in the next firing. The procedure followed in analyzing a practice (described in TM 4–235 (now published as TM 2160–35)) is designed to determine unusual personnel errors, accidental errors, and the systematic error of each gun. The mean of the accidental errors is used to determine the developed armament probable error of the battery. Since all other causes of a systematic error are measured with a reasonable degree of accuracy and corrections are applied to compensate for their effect, the entire systematic error of a firing is considered to be caused by one factor, an erroneous assumption of muzzle velocity. The range error is converted into muzzle velocity effect and this is applied as a correction to the assumed muzzle velocity. The velocity thus determined is that one which theoretically would have brought the center of impact accurately on the target if it had been used in the calculation of firing data. W. D., C. A. C. Form No. 25 (Matériel and Powder Report) is provided for keeping a record of the muzzle velocity at standard temperature apparently developed by each gun. This serves as valuable reference material in preparation for future firings.

SECTION V

CALIBRATION

■ 114. GENERAL.—*a*. When it has been observed that the centers of impact of shots from individual guns of a battery do not fall close together, some attempt should be made to determine corrections that when applied to individual guns will cause them all to attain the same range. This is called "calibration."

b. It seems reasonable to ascribe the cause of unequal ranges attained by guns using the same ammunition mostly to erosion and to treat the effects as differences in developed muzzle velocity. Although it is quite probable that the differences in range are not due entirely to differences in the developed muzzle velocity, the problem is considered as one of detemining the velocity differences of the individual guns and applying corrections that will eliminate or at least lessen the effects.

■ 115. DETERMINATION OF VELOCITY DIFFERENCES.—*a*. Velocity differences are obtained from records of actual firings. The deviations of all shots are determined, stripped of all known errors and corrections, and the resulting center of impact for each gun calculated. The deviations of the centers of impact are then converted into muzzle velocity effects by reference to the firing tables, and the velocity differences thus obtained. One gun of the battery is then selected as the standard with which to compare the other guns. This gun is called the "reference piece" and will have no calibration correction. The piece selected as the reference piece should be that one which will cause the resulting corrections to be the minimum in number and magnitude. The other guns are referred to as "test pieces."

b. W. D., C. A. C. Form No. 25 gives the muzzle velocity at standard temperature developed by each gun for any previous firing. This source of information will be the usual one used to determine velocity differences. Lacking this source of information, it may be necessary to fire a special calibration problem.

c. The centers of impact of individual pieces are subject to accidental variation, and as are shown in paragraph 129, the range difference between the centers of impact of any two guns which is as likely as not to be exceeded is

$$\frac{\sqrt{2}\times \text{P. E. (yards)}}{\sqrt{n}}$$

where P. E. represents the probable error of a single shot and n represents the number of shots from each gun. It is evident that on converting range probable error in yards to a muzzle velocity probable error, the probable error of the developed muzzle velocity determination, calculated as in *a* above, corresponds to

$$\frac{1.4\ \text{P. E. (f/s)}}{\sqrt{n}}$$

d. Within limits the value of the calibration data varies with the number of shots considered. Therefore, as many shots of the firing as possible should be used in determining the centers of impact. However, if a shot is erratic, due to warming-up effect or some other cause, it should be disregarded. (See pars. 102*g* and 151.)

e. The centers of impact of the reference piece and the test pieces are subject to accidental variation, so that a single determination of the velocity differences will not be as valuable as the mean of several determinations. If the number of shots considered in each case is not the same, the mean of the several determinations should be a weighted mean; all else being equal, the weight given to any determination should be proportional to the square root of the number of shots considered. (See par. 129.) If the number of shots in each case is about the same, however, calculations are simplified and results are approximately the same if the weight given to any determination is directly proportional to the number of shots. However, other factors, such as adverse weather conditions or poor behavior of ammunition, should also be taken into account. The weight given to a calibration firing should be not less than two and not greater than four times that given to a target practice of an equal number of rounds. Since the calibration correction may be expected to vary with the range, the determinations grouped together to give a mean should be those made at approximately the same range. Complete records of all calibration data and velocity differences should be kept on file.

■ 116. APPLICATION OF CORRECTIONS.—*a*. After the velocity differences have been determined, they must be transformed into calibration corrections that can be applied to individual guns. They cannot be applied to each gun in terms of muzzle velocity. They are usually determined as angles of elevation varying with the range, and may be applied in one of several ways. For example, on guns using the M5 data transmission system, a variable correction in mils of elevation could be taken from a chart and applied on the differential of the transmitting unit in the plotting room; while on guns having an angle of site adjustment on the sight, a variable correction could be applied to the angle of site. Either of these methods gives a range correction whose magnitude varies with the range in much the same manner as a muzzle velocity correction would. Or the muzzle velocity differences may be converted into range differences at mean range and applied as flat elevation corrections; that is, by displacing the range disks or the elevation quadrants. This method gives a range correction whose magnitude varies inversely with the range.

b. The choice of methods depends on the results of the determinations of velocity difference. If, as is expected, those results show a fairly constant velocity difference over wide limits of range, variable elevation corrections should be used. If the velocity difference varies as the range changes, a flat elevation correction may be more suitable. The battery commander should consider all elements of the problem and select the method that will give him the simplest solution and still give satisfactory accuracy for all guns throughout the field of fire.

■ 117. FIRING.—*a*. In general, the preparation of a battery for calibration firing will be the same as that made before any other firing. (See FM 4–20.) The adjustment of range disks and quadrants should be given special attention and, if possible, the pointing in elevation should be checked by gunner's quadrant for every shot. Observing instruments should be checked so that there may be no doubt as to their adjustment.

b. The target used will be a *fixed* target, usually an anchored pyramidal target. Its position should be checked by replotting at frequent intervals.

c. Recorded data should show a full account of meteorological conditions, the seating of the projectile for each shot, the length of recoil for each shot, the length of time each powder charge is in the powder chamber, and any other data that may be useful in case a question should arise as to whether or not any apparently erratic shot should be disregarded in computing the calibration data. If a chronograph is available, it should be used and its records considered in making the analysis of the firing.

d. Calibration shots should be fired when meteorological conditions are as nearly normal and stable as practicable. The wind especially should be of low velocity and steady.

e. Observation of fire should be conducted with all means available. The observations should be made by as many plotting and spotting systems as can be brought to bear, and the result accepted as true should be a weighted mean of all determinations, the weights being assigned according to the relative reliabilities of the various systems used. Camera records taken from a vessel near the target will have great value. Generally those systems of spotting in which the deviation from the target is measured will be more reliable than those in which the splash is located by azimuths.

f. The guns should be fired in rotation so that slow changes in ballistic conditions will affect all alike. The rate of fire should be as rapid as is consistent with accuracy, so that conditions will have less time to change.

g. Adjustment of fire during calibration firing will be exceptional, but should be made when it will improve the accuracy of spotting. (See par. 144.)

■ 118. DATA FROM TARGET PRACTICES.—Round for round, the calibration data gained from target practices are less valuable than data gained from properly conducted calibration firings, but target practices give a source of information that should not be ignored. Space is provided on W. D., C. A. C. Form No. 25 for a systematic recording of calibration data.

These data should be filled in on the individual reports after each target practice and should be collected in appropriate form in the battery emplacement book.

■ 119. EXAMPLES.—*a*. A battery of two 12-inch guns, M1895, on barbette carriage M1917, firing the 975-pound projectile (FT 12–F–3), was fired for calibration at a fixed target at a range of 29,000 yards. All shots were fired with the same quadrant elevation. The center of impact of No. 1 gun was "short 150 yards" and that of No. 2 gun was "over 50 yards." Taking No. 1 gun as the reference piece, what is the indicated velocity difference?

Solution: The center of impact of No. 2 gun is 200 yards beyond that of No. 1 gun. From table *Fa*, part 2, of the firing tables, the velocity difference is

$$10+\frac{11}{189}\times 10=11\ f/s \text{ (to the nearest } f/s\text{)}.$$

b. A battery of four 155-mm guns, M1918MI, using HE shell, Mk. III, with fuze, short, Mk. IV* (FT 155–B–4), was fired for calibration at a fixed target at a range of 13,000 yards. The normal charge was used, a muzzle velocity of 1955 *f/s* was assumed, and eight rounds were fired from each gun. The calibration data obtained from this firing, as shown by W. D., C. A. C. Form No. 25, are given in the tabulation below. What are the indicated velocities for this firing?

Solution: See last line of tabulation.

Tactical number of piece	1	2	3	4
Zone	----------	----------	----------	-------
Number of shots	8	8	8	8
Average actual range	13,000	13,000	13.000	13,000
Assumed MV, 70°	1,955	1,955	1,955	1,955
Deviation of CI (yards)	+35	+10	+55	+90
Deviation in *f/s* MV	+6	+2	+9	+14
Developed MV, 70°	1,961	1,957	1,964	1,969

As a result of these determined velocities, No. 1 gun is selected as the reference piece, and the following calibration corrections (foot seconds) are made:

Gun No.	1	2	3	4
Correction	0	+4	−3	−8

c. The battery referred to in *b* above later fired a target practice at a mean range of 12,480 yards (for record fire) using calibration corrections based on the velocity differences determined in the calibration firing. The additional calibration data tabulated below were obtained from this firing. What velocity differences were developed during this firing?

Solution:

Tactical number of piece	1	2	3	4
Zone				
Number of shots	8	7	7	8
Average actual range	12,480	12,480	12,480	12,480
Assumed MV, 70° [1]	1,961	1,957	1,964	1,969
Deviation of CI (yards)	−12	−42	−32	+38
Deviation in *f/s* MV	−2	−7	−5	+6
Developed MV, 70°	1,959	1,950	1,959	1,975

[1] The assumed MV for each gun is determined by subtracting the calibration correction from the assumed MV of the reference piece.

Velocity differences developed during this firing are as follows:

Gun No.	1	2	3	4
	Ref.	−9	0	+16

d. The record kept by this battery over a period of time, by collecting data similar to those already shown, is as follows:

Record of velocity differences

[Normal charge]

Tactical No.		1		2		3		4	
Register No.		375		148		221		47	
Date	Range	Shots fired	Velocity difference	Shots fired	Velocity difference	Shots fired	Velocity difference	Shots fired	Velocity difference
Calibration									
Feb. 10, 1935	13,000	8	Ref.	8	−4	8	+3	8	+8
Target practices									
Feb. 17, 1935	12,480	8	Ref.	7	−9	7	0	8	+16
Feb. 20, 1935	11,920	7	Ref.	8	−7	7	−5	8	+10
Mar. 12, 1935	12,200	10	Ref.	10	−10	10	+2	10	+14
June 22, 1935	10,800	8	Ref.	7	−9	7	+4	8	+10

What mean velocity difference may be determined from these data?

Solution: There can be no exact solution to such a problem; the following is given as a satisfactory solution only. The first decision to be made entails a choice of the firings to be used in the computation. Since all the firings were held at approximately the same range it was decided that all should be used.

Next comes the matter of weighting factors. The following facts were considered in making the decision: The rate of fire of this type of gun ordinarily makes pointing not as accurate as is desired for calibration purposes, but the results of these practices were excellent, the shots were well grouped and close to the target, showing that the pointing was good enough to permit use of the data. It was therefore decided to give the calibration firing a weight of three and each target practice a weight of one. In addition, each practice was weighted according to the number of rounds fired because the number of rounds in each case is approximately the same. The calculation of the weighted means may now be made as follows:

For tactical No. 2, register No. 148:

$$\frac{(3\times 8\times(-4))+(7\times(-9))+(8\times(-7))+(10\times(-10))+(7\times(-9))}{(3\times 8)+7+8+10+7}=\frac{-378}{56}=-7f/s$$

For tactical No. 3, register No. 221:

$$\frac{(3\times 8\times(+3))+(7\times(-5))+(10\times(+2))+(7\times(+4))}{(3\times 8)+7+7+10+7}=\frac{+85}{55}=+2f/s$$

For tactical No. 4, register No. 47:

$$\frac{(3\times 8\times(+8))+(8\times(+16))+(8\times(+10))+(10\times(+14))+(8\times(+10))}{(3\times 8)+8+8+10+8}=\frac{+652}{58}=+11f/s$$

CHAPTER 10

PROBABILITY OF ERRORS

SECTION I

GENERAL

■ 120. GENERAL.—Previous discussion of probable error has been based on the firing of a limited number of shots, while in reality the true value of the probable error could be determined only after firing an infinite number of shots. The results obtained by applying the rules of probability and theory of errors to artillery firing obviously are not precisely correct but they approach the practical results needed. In this chapter, an attempt is made to discuss briefly the more academic theory of errors in its relation to gunnery.

SECTION II

MATHEMATICS OF PROBABILITY

■ 121. GENERAL.—Probability is a branch of mathematics that permits the reckoning of the likelihood of a thing concerning which information is not complete. It may deal with the occurrence or nonoccurrence of an event, past, present, or future. It may deal with the truth or falsity of a statement or a conclusion. It furnishes a guide to sound reasoning when chance takes the place of certainty.

■ 122. NUMBERS USED.—When a thing is certain, its probability is represented by the number 1 (unity); when it is impossible, its probability is represented by 0 (zero); and when it is neither certain nor impossible, its probability is represented by a number greater than zero and less than one. If a thing is as likely as not, its probability is one-half. As its likelihood increases, its probability approaches unity. As its likelihood becomes less, its probability gets closer to zero. If the odds in favor of a thing are three to one, its chances are three out of four, or its probability is three-fourths. If a thing has one chance in ten, its probability is one-tenth.

■ 123. RULE FOR ADDITION.—*a*. Two things are mutually exclusive when the occurrence of the first prohibits the occurrence of the second, and the occurrence of the second prohibits the occurrence of the first. If the probability of the first thing is P and the probability of the second thing is Q, the two things being mutually exclusive, then the probability of either the first or the second is $P+Q$. This is the rule for the addition of probabilities and may be extended to include any number of mutually exclusive events or conclusions.

b. For example, the probability that a shot will hit the side of a ship is 0.08, and the probability that it will hit the deck is 0.13. What is the probability that it will hit either the side or the deck?

Solution: A shot cannot hit both the side and the deck, so that the two events are mutually exclusive. The rule for addition may therefore be applied, and the required probability is the sum of 0.08 and 0.13. $0.08+0.13=0.21$.

■ 124. RULE FOR MULTIPLICATION.—*a*. Suppose the occurrence of event A consists of the joint occurrence of both event B and event C. Then the probability of A is the product of the probability of B times the probability that if B occurs C will occur also. If the occurrence or nonoccurrence of B has no effect upon the probability of C, the rule reduces to the probability of A is equal to the probability of B times the probability of C. This is the rule for the multiplication of probabilities and may be extended to include any number of contributing events.

b. An example is found in paragraph 101*g* and in figure 24. The probability that a shot will hit a particular rectangle is the product of the probabilities of its hitting both the proper range strip and the proper lateral strip.

c. As another example, suppose that the probability that a shot will be a hit in range is 0.26 and that of its being a hit in direction is 0.68. Then the probability of the shot's being a hit, that is, good in both range and direction, is equal to $0.26 \times 0.68 = 0.1768$ or 18 percent.

■ 125. Rule of Repetitions.—*a*. Suppose that the event A consists of the occurrence of event B repeated n times and event C repeated m times, the events B and C being independent of one another and their order of occurrence immaterial. Let the probability of B be P and that of C be Q. Then, under the rule for multiplication, the probability of A with any one particular arrangement of B's and C's is equal to ($P \times P \times P \times$—to n factors) $\times$ ($Q \times Q \times Q \times$—to m factors) $= P^n Q^m$. All possible arrangements must be considered. The number of arrangements may be found by calculating the number of permutations of $m+n$ things of which m are of one kind and n of another. This number is $\frac{(m+n)!}{m!\,n!}$ read "factorial $(m+n)$ divided by the product of factorial m times factorial n"). Since the B's and C's are independent, one arrangement is as likely as another. Since the various arrangements are mutually exclusive, their probabilities may be added. In this case, repeated addition is equivalent to multiplication. Therefore

$$\text{the probabililty of } A = \frac{(m+n)!}{m!\,n!} P^n Q^m$$

The same reasoning applies when the number of contributing events is greater than two. The factorial in the numerator of the coefficient is the factorial of the sum of the numbers of repetitions, while those in the denominator are the factorials of the separate numbers.

b. For example, when a number of shots are fired at a target, the probability of getting *exactly* n hits may be computed by this rule. The calculations are shown below for the case when the number of shots is four and the probability of hitting on any single shot is 0.18.

Note.—Take 0! as equal to 1.

Number of hits	Substitution in formula	Reduction	Probability
0	$\frac{4!}{0!\ 4!}\times 0.18^0\times 0.82^4$	$1\times 1\times 0.45$	0.45
1	$\frac{4!}{1!\ 3!}\times 0.18^1\times 0.82^3$	$4\times 0.18\times 0.55$	.40
2	$\frac{4!}{2!\ 2!}\times 0.18^2\times 0.82^2$	$6\times 0.03\times 0.67$	.12
3	$\frac{4!}{3!\ 1!}\times 0.18^3\times 0.82^1$	$4\times 0.01\times 0.82$	.03
4	$\frac{4!}{4!\ 0!}\times 0.18^4\times 0.82^0$	$1\times 0.00\times 1$	.00
Total			1.00

In each substitution, n is the number of hits, m is the number of misses, $m+n$ is equal to four (the number of shots fired), P is equal to 0.18 as given, and Q is equal to $1-P$ since $P+Q=1$ (one or the other of the two events must occur). The sum of the computed probabilities equals 1. This is because all possible outcomes have been considered.

SECTION III

CURVE OF ACCIDENTAL ERRORS

■ 126. GENERAL.—*a*. For most practical purposes, the rough ideas of dispersion conveyed by the dispersion ladder are good enough, but calculations are made easier and more refined by the use of the curve of accidental errors (fig. 28). To show

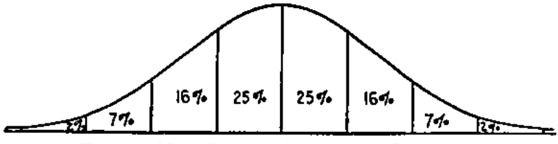


FIGURE 28.—Curve of accidental errors.

the relation between the curve and the ladder, a part of the horizontal axis has been divided into eight equal parts and at each point of division a perpendicular has been erected. This divides the major part of the area included between the curve and the axis into parts whose areas are very nearly proportional to the numbers 2, 7, 16, 25, etc., of the dispersion ladder.

b. The division of the area included between the curve and the horizontal axis may be carried further. The whole of this area is made equal to unity, and then the part of this area included between any two perpendiculars is equal to the probability that a shot will fall between the two points in the field of fire represented by the two points at which the perpendiculars are erected. To determine the area under the curve and included between two perpendiculars and the axis, table II–A (appendix VI) is used.

c. When two perpendiculars are erected at equal distances from the center of dispersion, as at *A* and *B* in figure 29, the area included between them will depend upon the ratio of the distance *OB* to the distance *OP* (the probable error), or

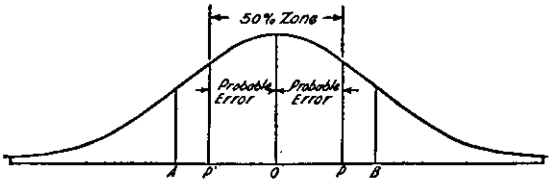


FIGURE 29.—The factor.

what is the same thing, the ratio of the distance *AB* to the distance *P'P* (the 50-percent zone). When this ratio is fixed, the area is fully determined. This ratio is called the "factor." In table II–A (appendix VI), values of the area are listed under the heading "Probability" opposite the corresponding values of the factor. In table II–B (appendix VI), values of the factor, or ratio, are listed opposite corresponding values of the probability or area. There is no essential difference between the two tables; the two arrangements are for greater convenience in making interpolations.

■ 127. USE OF CURVE IN COMPUTATION OF PROBABILITY OF HITTING.—*a. Center of dispersion on center of target*.—The center of dispersion is placed on the center of a target as shown in figure 30. The dimension of this target in the direction of the gun–target line is 120 yards, and its lateral dimension is so great that no shots will be expected to miss it in direction. The probable error in range is 100 yards. Compute the probability of hitting with a single shot, using both the dispersion ladder and table II-A (appendix VI) and compare the results.

Solution with dispersion ladder: The 50-percent zone is twice the probable error, or 200 yards. The target occupies 120/200, or 60 percent of the 50-percent zone. It will there-

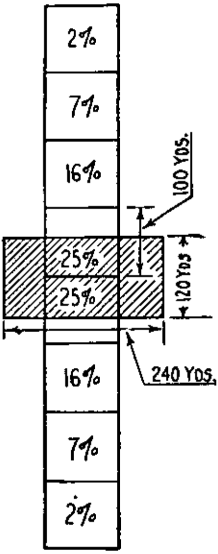


FIGURE 30.—Center of dispersion on center of target.

fore be expected to contain 60 percent of 50 percent of the shots, or 30 percent.

Solution with probability tables: The factor is equal to 120/200 (or 60/100) = 0.60. According to the table, the probability corresponding to this factor is 0.314, or 31.4 percent.

Comparison of results: The two results do not check exactly. The use of the probability table is equivalent to finding the area under a part of a smooth curve as in the upper part of figure 31, while the use of the dispersion ladder corresponds to finding the area under a broken line as in the lower part of figure 31.

b. Center of dispersion not on center of target.—(1) When it is required to find the probability of hitting between two points

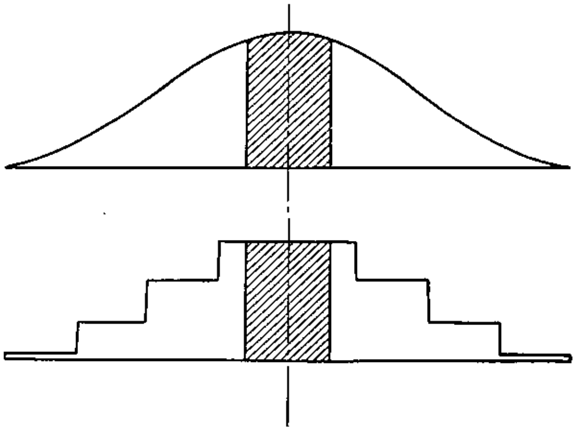


FIGURE 31.—Dispersion ladder and curve of accidental errors.

which are not symmetrical with respect to the center of dispersion, as between the points *C* and *D* in figure 32, it will be necessary to use the table twice. The shaded area *ABDC*

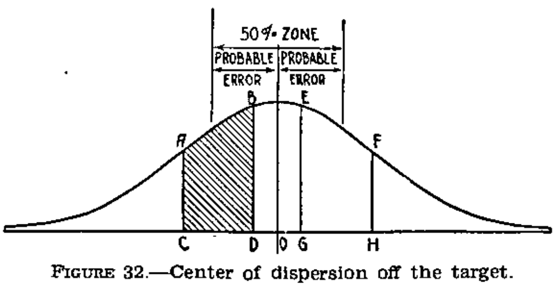


FIGURE 32.—Center of dispersion off the target.

is required. The ratio of the distance *CO* to the probable error will give, through the use of the table, the area of the figure *ABEFHC*. The ratio of the distance *DO* to the probable error will give, by the table, the area of *BEGD*. If the smaller area is subtracted from the larger and the remainder divided by two, the result will be the required area which is equal to the required probability.

(2) As an example, take all conditions the same as in example under *a* above, except that the center of dispersion is 150 yards beyond the center of the target. The situation will then be as shown in figure 33. Find the probability of hitting.

Solution: The two edges of the target will be 90 yards and 210 yards, respectively, from the center of dispersion. The first factor is $F_1=210/100=2.10$. The corresponding probability is $P_1=0.843$. The second factor is $F_2=90/100=0.90$.

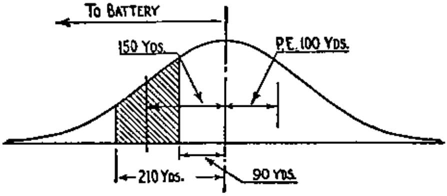


FIGURE 33.—Illustration for example, paragraph 127*b*.

Then $P_2=0.456$. The probability of hitting is equal to $P=\frac{1}{2}(P_1-P_2)=\frac{1}{2}(0.843-0.456)=0.193$, or 19.3 percent.

NOTE.—In appendix II, instructions are given in the use of an alinement chart for the solution of problems of this nature. It may be used in place of tables II–A and II–B (Appendix VI).

■ 128. COMPUTATION OF PROBABILITY IN OTHER OPERATIONS.—The method of calculating the probability of a shot's falling between certain points given in paragraph 127 is equally applicable to the calculation of the probability that any variable distributed in the same manner will take on a value between specified limits. It has been mentioned in paragraph 104 that accidental errors are usually considered as distributed in this way. In fact, the study of the distribution of accidental errors preceded the study of dispersion in gunfire and governed the course taken by the latter study. The accidental errors with which this manual is concerned are those occurring in spotting, position finding, calibration, and pointing.

SECTION IV

DISTRIBUTION OF ERRORS

■ 129. DISPERSION OF CENTERS OF IMPACT.—*a*. If a large number of shots were fired in several short series, the centers of impact of the several series would be scattered in a manner similar to the scattering of the individual shots but over a smaller area. The curve showing the dispersion of centers of impact would be higher at the peak and its main part would cover less horizontal distance than the curve showing the dispersion of single shots. In figure 34, curve *A* shows the dispersion of separate points of impact and *B* shows the distribution of centers of impact of groups containing four

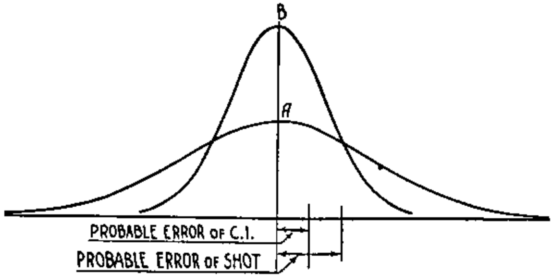


FIGURE 34.—Distribution of centers of impact.

shots each. It can be shown that the probable error of the center of impact of a group containing n shots is equal to the gun probable error divided by the square root of n. That is,

$$r_c = \frac{r_g}{\sqrt{n}}$$

where r_c is the probable error of the center of impact, r_g is the gun probable error, and n is the number of shots in the group.

For example, the gun probable error is 72 yards. The probable error for the center of impact of a group of nine shots would then be $72/\sqrt{9}=72/3=24$ yards. This means that the center of impact is as likely as not to be more than 24 yards from the center of dispersion.

ADJUST BY C.I.? AND ITS P.E.?

ONLY FOR BRACKETING, 1
[illegible]
OR AXIAL

P.98

b. The difference between the deviations of two successive shots is a quantity whose distribution may be determined if the distribution of shots is known or assumed. If the shots are normally distributed, the differences between successive deviations will also be normally distributed. If r_g denotes the gun probable error used to measure the dispersion of individual shots, then the probable error that measures the distribution of these differences will be equal to $r_g\sqrt{2}$. In the same way, differences between the range deviations of successive salvo centers of impact are normally distributed, the corresponding probable error being equal to the probable error of the salvo center multiplied by the square root of 2. For example, the distance between the centers of impact of two successive salvos in *a* above that is as likely as not to be exceeded is $1.4\times24=33.6$ yards.

■ 130. COMPOUND ERRORS.—*a*. A compound error is the result of two or more independent errors acting jointly. Thus, personnel errors and armament errors together determine the point of impact. Dispersion and the spotting error together determine the spotted deviation. If each contributing error is distributed normally, that is, can be shown by a curve like that in figure 28, if the contributing errors are independent of one another, and if their values are added algebraically to determine the value of the resultant error, then the resultant error is normally distributed and its probable error is equal to the square root of the sum of the squares of the probable errors of the contributing errors.

b. The spotting error is not independent of the magnitude of the deviation, so that in compounding it with other errors the second of the conditions listed in the rule above is not fulfilled. It is permissible to assume that it is independent, simply for the purpose of making an approximate calculation. After the further assumption that spotting errors are normally distributed, the distribution of *spotted* points of impact as distinguished from *actual* points of impact may be calculated by this rule. The distribution of spotted points of impact is normal and its probable error is equal to

$$r_s=\sqrt{r_g^2+r_o^2}$$

In this equation, r_s is the probable error showing the distribution of the spotted points of impact, r_g is the probable error of gun dispersion, and r_o is the probable error of observation.

■ 131. DEVELOPED ARMAMENT PROBABLE ERROR.—The probable error in gunfire, as defined in paragraph 108, is the distance from the center of impact of a large number of shots to the point of impact which is as likely as not to be exceeded on any one shot, and the probable error is equal to the mean error of a series of shots multiplied by 0.845. This leads to the definition of developed armament probable error as equal to the mean armament error multiplied by 0.845. For example, take the series of four shots considered in paragraph 102*d*, whose range deviations are +180, —60, —45, and +73 yards, respectively, and whose lateral deviations are —42, —18, +24, and —22 yards. The center of impact was found to lie 37 yards beyond the target and 14.5 yards to the left of it. The armament errors of these shots are then as follows:

Shot	Armament error, range	Armament error, lateral
1	$+180-37=+143$ yards	$-42-(-14.5)=-27.5$ yards.
2	$-60-37=-97$ yards	$-18-(-14.5)=-3.5$ yards.
3	$-45-37=-82$ yards	$+24-(-14.5)=+38.5$ yards.
4	$+73-37=+36$ yards	$-22-(-14.5)=-7.5$ yards.

The mean range error (arithmetical mean, that is disregarding algebraic signs) is $(143+97+82+36)/4=358/4=89.5$ yards. The mean lateral error is equal to $(27.5+3.5+38.5+7.5)/4=77/4=19.25$ yards. The developed range probable error is equal to $89.5\times0.845=75.6$ yards. The developed lateral probable error is equal to $19.25\times0.845=16.3$ yards. (The developed probable error is not the probable error of the dispersion but merely an indication of it, because the impacts of a large number of shots must be considered to determine the latter.)

CHAPTER 11

ADJUSTMENT OF FIRE

SECTION I

GENERAL

■ 132. GENERAL.—*a*. Adjustment of fire is a process of applying such adjustment corrections to the firing data as will place the center of dispersion on the target and keep it there. It is a necessary process because, even after the most careful preparations for firing, the center of dispersion initially may not be on the target. Adjustment of fire is a continuous process throughout all firing, for even though the center of dispersion has been placed on the target, varying conditions may cause it to drift away. Occasions may arise where, due to fog, smoke screens, darkness, or other causes, observation will be fleeting or impossible. Under these circumstances, such information concerning deviations as is obtained, even though that information is meager, should be used in accordance with the principles of adjustment hereinafter given. Experience has shown that fire can be made destructive at short and medium ranges without adjustment, so that even though observation of fire is impossible there should be no hesitancy in opening fire at such ranges with the best available data.

b. The fact that a hostile vessel may be under fire but a short period of time requires that fire is not delayed to place the center of impact on the target, but that it is continuous when there is a reasonable certainty that the target is in the hitting area; that is, within three probable errors of the center of dispersion (or center of impact). In certain cases where ranges are so extreme that the probability of hitting is small or when the conservation of ammunition is

FORK AND A HALF

imperative, fire may be conducted with deliberation. Advantage should then be taken of the slow rate of fire to apply such corrections as will best insure obtaining hits on the target. Under such conditions, once the hostile vessel is brought within the hitting area, bursts of rapid fire at intervals will prove more effective than continuous fire and corrections may be applied after each burst.

c. The conditions which must be met in devising rules for the adjustment of fire are as follows:

(1) The rules must deal with accidental errors.

(2) They must deal with systematic errors, both constant and varying.

(3) They must be flexible in order that they may meet all tactical situations which may arise.

(4) They must give rapid adjustment without undue waste of ammunition.

(5) They must be simple, easy to remember, and easy to apply.

d. Instructions for the adjustment of fire have been prepared with the object of standardizing procedure and of placing in concrete form the results of experience. It is not intended that they shall be absolutely binding on the officer adjusting fire nor that they shall limit his initiative, but that they shall serve as a guide to the procedure which under average conditions will be most effective. Any proper combination of the various methods given may be made if such combination is desirable.

■ 133. Basic Assumptions.—In adjustment of artillery fire it is assumed that—

a. Either measured deviations or sensings of impacts (or centers of impact of salvos) are obtainable.

b. The individual adjusting the fire has a general knowledge of the effects in range and direction of nonstandard conditions as given in the firing tables and is familiar with the application of probability and the theory of errors to artillery firing.

■ 134. Basic Principles.—The rules for adjustment of fire as laid down in this chapter are general in nature. It is essential that the person responsible for the adjustment of fire be familiar with the basic principles stated below.

a. Proper preparation for firing by eliminating systematic errors, especially those which vary progressively, and by placing the center of impact near the target, will prevent as far as possible the shifting of the center of dispersion and will place it near the target with a saving of time and ammunition. This places the adjustment of fire on a sound foundation.

b. The practical unit of measure for use in adjustment of fire is the probable error. The best value of the probable error is obtained from a study of previous firings of the battery. If those records are not available, the firing table value should be used.

c. Due to accidental errors, a shift of the center of impact, as indicated by a few salvos or a short series of shots, may be expected. The shift is due to accidental errors alone, and occurs even though the center of dispersion is not shifting. The amount of natural shift which may be expected depends upon the size of the probable error being developed and upon the number of shots or salvos considered in determining the center of impact.

d. The probable error of the center of impact of a salvo is equal to the gun probable error divided by the square root of the number of shots in the salvo. The distance between the centers of impact of two successive salvos that is as likely as not to be exceeded is 1.4 times the salvo probable error. Hence, the natural shift of the center of impact of a four-gun salvo which is as likely as not to be exceeded is about one gun probable error.

e. In general, a correction should be based on a consideration of the total number of rounds which have been fired under the same conditions. The larger the number of rounds considered, the closer will the correction approximate the deviation of the center of dispersion. On the other hand, if too many rounds are considered, conditions may have changed since the earlier rounds were fired and the center of dispersion may have shifted. Decision as to the number of rounds to consider requires the exercise of sound judgment on the part of the officer adjusting fire.

f. Unless there is a strong indication that the center of dispersion is shifting, no correction should be applied as a result of fewer shots than were considered in the determination of the previous correction.

g. Over adjustment should be avoided. As long as there is a reliable indication that the center of dispersion is reasonably close to the target, it is better to wait until a sufficient number of shots have been spotted to give a good determination of the correction required than to attempt to outguess the natural shift of the center of impact.

h. Mistakes may be made in the application of adjustment corrections. This fact alone should prohibit the application of small and frequent corrections.

i. The object of adjustment of fire has been stated as "to place the center of dispersion on the target and keep it there." In view of the fact that the exact location of the center of dispersion is never known, the question arises as to what minimum deviation of the center of impact calls for an adjustment correction. No absolute value can be stated. A decision in a particular case should be based upon a knowledge of the probable error of the position-finding system, armament probable error, probable error of the spotting system, amount of confidence which may be placed in the preparation for firing, requirements of the tactical situation, state of training of personnel, and the principles outlined herein. In general, fire will be very effective if the center of dispersion has been placed within one probable error of the target. The application of a correction of less than one-half probable error is never warranted.

j. Having decided that a correction is necessary, it should be made to the least 1/10th of 1 percent of the range or nearest 10 yards.

k. Corrections based on measured deviations are more likely to give quick adjustment than those computed in any other manner.

l. When measured deviations are not available, corrections should be based on the sensing of overs and shorts. Even if adjustment by the use of measured deviations is the standard method, *the battery should be prepared to continue the adjustment at any time by means of sensings*. This requires that a bracketing adjustment chart be kept available for immediate use in case of necessity. In order that the chart be adequate for all ranges, a probable error for medium range should be used in its construction. Sensings of the last few shots, with the correction on which they were fired, can be taken from the fire adjustment board.

m. An equal number of overs and shorts obtained with the same adjustment correction is a good indication that the center of dispersion is close to the target.

n. During fire for effect, from one to three additional salvos may be fired during the interval between the firing of the last salvo considered and the firing of the first salvo with the new correction applied. Care should be taken that a correction ordered is not considered in the further adjustment of fire until the fall of the salvo on which it first took effect has been spotted. A careful plan and considerable training are required to insure that no such mistake is made.

o. A bold correction should be made, if necessary, to bring the center of impact close to the target as early in the firing as possible. This action is justified by the fact that small deviations can be measured more accurately than large ones. This should increase the accuracy of spotting.

LATERAL

5 *p.* Experience has shown that very reliable adjustment data may be obtained as a result of deliberate trial fire which has been preceded by careful preparation for firing. Data obtained in this manner should not be lightly discarded, particularly when subsequent fire takes place under approximately the same conditions.

q. Fire should not be continuous if the target is not within the hitting area.

r. Occasionally an erratic or wild shot will be fired. A shot should be considered wild when its impact is more than four developed armament probable errors or, in the absence of this information, more than ~~four~~ 6 firing table probable errors from the center of impact. A wild shot should be disregarded in determining an adjustment correction.

s. The rules for adjustment are designed to take care of the normal cases where the systematic errors are nearly constant and the distribution of accidental errors is reasonably in accordance with the laws of probability. An extraordinary circumstance may occasionally arise to cause the rules to be ineffective. In such a case the cause must be located and removed.

■ 135. SHIFTING CENTER OF DISPERSION.—*a*. The idea of a fixed center of dispersion is based on the thought of minor variations in the things that affect the range attained by the projectile, variations that come and go quickly, leaving no real change in conditions. There are times, however, when the fall of shots can be explained only as the result of a fundamental change in conditions or else as a freakish distribution of errors. At such times, the shots have a tendency to get farther and farther from the target and adjustment seems impossible. An accurate knowledge of all things concerning the battery is then very important. If there is reason to believe that conditions are actually changing, then a predicted adjustment correction may be justified, but this reason must be real and tangible. If a predicted adjustment correction is made when it is not warranted, it is as likely as not to put the next shot away from the target and on the opposite side.

b. Even though a predicted correction is out of the question, it still remains to be decided just how many of the past shots will be considered in making the next adjustment correction. Since the accuracy of determination of a correction increases approximately as the square root of the number of shots considered, there is not a great deal to be gained by going too far back in any series, especially if there is a reasonable indication that conditions have changed. If there is no indication of a change, that is, if the scattering appears to be due to normal dispersion, then the larger the number of shots considered, the better the correction should be. It is because of the assistance it gives in deciding these difficult questions that the graphical fire adjustment board is so valuable.

c. A similar question arises when trial shots are fired but shooting is then delayed. When firing starts again, should the trial shot correction be used or should trial shots be fired again? Except when no meteorological data have been obtained, the firing of a second series of trial shots is hardly ever justified. If there is reason to believe that the muzzle velocity is well established and that most of the trial shot correction is needed because of undetermined meteorological conditions, then it is doubtful if a correction will be good for any extended period of time, and a second series of trial shots may be warranted.

■ 136. METHOD OF APPLYING CORRECTIONS.—*a. Range.*—When the range to the target is changing, the correction necessary to maintain the center of dispersion on the target cannot be expected to remain a constant flat value. Corrections under such conditions should be applied, not as flat corrections but rather as variables, which, if possible, follow the same law as that governing the magnitude of the existing sytematic error. The standard practice, except in 3-inch rapid-fire batteries, is to apply the range correction as a percentage of the range. This method is convenient and by experience it has been found to be a reasonably accurate method of maintaining the center of dispersion on the target.

b. Direction.—It is convenient for low angle fire to apply corrections in direction as flat angular corrections. This results in the application of linear corrections which will vary in proportion to the range. For high angle fire, this correction should be applied as a function of the quadrant angle of elevation. This is due to the fact that the most probable cause of the deviation of the center of impact, in direction, from the point fired at, is an incorrect determination or assumption of the cross wind effect. For high angle fire, the cross wind effect varies approximately as the quadrant angle of elevation which increases as the range decreases. Corrections for direction should be made on the deflection board but for mobile artillery they may be applied on the panoramic sight.

■ 137. DANGER SPACE.—*a.* Danger space is defined as that portion of the range within which a target of given dimensions would be hit by a projectile with a given angle of fall. It is that area indicated by projecting the target on the surface of the earth or water by lines parallel to the line of fall of the projectile. Theoretically the greatest probability of hitting will exist when fire is adjusted so that the center of dispersion is on the center of the danger space. Ordinarily the observing point on a target is somewhere on the center line of the ship. Obviously this will not coincide with the center of the danger space, and in rare cases it may be deemed necessary to apply a correction to firing data to compensate for this error. But no such correction should be included in a fire-adjustment correction determined from observation of fire. If the firing is at an actual ship, all shots which come within the danger space are hits and all shots which hit the water over the target are in fact over the danger space. The same is practically true of the shorts. In this case, if the overs and shorts are equalized by adjustment, the center of dispersion will probably be on the center of the danger space. It is obvious that to apply a danger space correction to the computed adjustment correction in this case would be improper.

b. In the calculation of firing data, the application of a danger space correction presents additional complications. Fortunately the correction is usually so small as to make its application unnecessary.

c. (1) Assume that it is desired to compute the danger space under the following conditions:

12-inch BC gun (FT 12–F–3).
Range, 12,000 yards.
Target, battleship broadside.
Observing point, center of target.
Height of target above water line, 10 yards.
Height of target below water line, 4 yards.
Beam of target, 32 yards.

(2) In figure 35, *OSTP* represents a vertical section through the target along the direction of fire. The distance *AB* along the surface of the water is the depth of the danger space. The lines *SB* and *AP* are parallel to the line of fall for the range 12,000 yards and cut the section of the ship at points *S* and *P*.

(3) From the firing tables, it is found that the slope of fall is 1 on 5.4. The height *MS* is 10 yards and the distance *MB* would then be $5.4 \times 10 = 54$ yards. In a similar manner, the distance $AN = 4 \times 5.4 = 21.6$ yards. The danger space $AB = AN + NM + MB = 21.6 + 32 + 54 = 107.6$ yards. The center of the danger space would be 53.8 yards from point *A*. The center

of the target is 21.6+16=37.6 yards from point *A*. The danger space correction would then be 53.8−37.6=16.2 yards. The computation of the danger space correction may be somewhat simplified as follows:

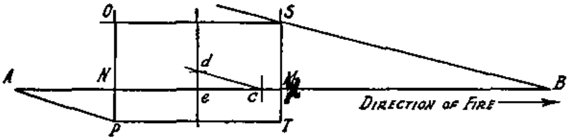


FIGURE 35.—The danger space.

Determine the distance *de*, where *d* is the geometrical center of the section *OSTP*, and multiply this value by the slope of fall.

$$de=\frac{10-4}{2}=3$$

and the danger space correction would be 5.4×3=16.2 yards.

■ 138. ADJUSTMENT OF SIMULATED FIRE.—The only manner in which thorough familiarity with the principles and rules governing the adjustment of fire may be attained is through regular and frequent training in their application. A satisfactory means of securing this training is by simulating fire and using the dispersion tape and scale described in appendix I. In using this device, the problem should be made as realistic as possible. It may readily be used at battery drill with the normal fire-control system, or it may be used for classroom instruction in which only the essential control features are provided.

SECTION II

METHODS OF ADJUSTING FIRE

■ 139. MAGNITUDE METHOD (DEVIATIONS MEASURED).—In this method of adjustment of fire, the magnitude and sense of the range deviation of each shot or salvo center of impact are spotted and the impacts are plotted graphically on the fire adjustment board. (See FM 4–15.) Corrections, mathematically as correct as known data on the center of dispersion will permit (except for slight inaccuracies in plotting) are then easily and quickly scaled from the plot. This method of fire adjustment has the great advantage of accomplishing its object with the least expenditure of ammunition and time. It is the standard method used by mortars and guns of 8-inch and larger caliber which have a slow rate of fire.

■ 140. **Bracketing Method (Deviations Sensed Only).**—When it is impossible to obtain more than the sense of the range deviation of each shot, it is necessary to use the bracketing method of adjustment in which the corrections are based on the relative number of overs and shorts. To facilitate the calculation of corrections, the bracketing adjustment chart is used (see FM 4–15). Obviously, the bracketing method requires the expenditure of more ammunition to obtain an original adjustment on the target than does the magnitude method. It is therefore standard for rapid fire batteries only. However, it is a very effective emergency system for larger caliber guns.

■ 141. **Classification of Fire.**—From the gunnery viewpoint, fire at naval targets may be divided into two classes; trial fire and fire for effect.

a. Trial fire has for its object the determination of a correction which, when applied, will cause the center of impact to be within three probable errors of the target. The deviation of the center of impact of the trial shots is the basis for determining the correction with which to begin fire for effect.

b. The object of *fire for effect* is the accomplishment of the tactical mission assigned the firing unit. This is done by placing the center of impact on the target and keeping it there in order that the maximum number of hits may be expected. Fire for effect should follow trial fire with as little delay as possible, otherwise the value of the correction determined as a result of trial fire will be reduced. When firing at naval targets, it may be difficult to spot, especially hits and overs. Firing will be most effective against a broadside target when approximately 40 to 45 percent of the shots are short of the water line of the target. If the target is bow on, this will be 20 to 30 percent. These percentages will vary with the size of the danger space as compared to the size of the zone of dispersion of the type of gun firing. Where

HITS CALLED OVERS → MORE

splashes both over and short of the target may be observed, practical equality of overs and shorts indicates that the center of impact is on the target.

■ 142. ADJUSTMENT FOR DIRECTION.—*a*. Adjustment for direction is based on observed lateral deviations. The corrections are applied concurrently with the corrections in range. When firing trial shots at a fixed point, the lateral correction to be applied when commencing fire for effect is based on the mean of the lateral deviations. If trial fire is conducted at a moving target and lateral corrections are applied during the firing, the correction determined as a result of the last shots should be used. If the target is moving uniformly, the sudden appearance of unusually large lateral deviations indicates personnel errors, and care should be exercised in applying a correction based on such deviations.

b. Lateral correction when employing case III fire may be made by methods analogous to the methods used in adjusting for range by the magnitude method. Because accidental errors in direction are usually small in comparison with the systematic errors, full corrections for the lateral deviations are generally justified. Unless there seems to be a wide lateral dispersion of shots wherein the salvo center of impacts appears to jump from one side of the target to another, corrections may well be based on the last shots spotted.

c. When employing case II fire, lateral corrections may be made by an axial observer located near the guns who calls the correction deflection. Another method, called "jumping splashes," is one in which the individual gun pointer makes the lateral correction. After the gun is fired, he traverses the piece to follow the target until the instant of impact, at which time he halts the piece and obtains a new sight setting by turning the line of sight of the instrument to the point of splash. This latter method is considered a less suitable one because it is believed that in service the gun pointer will be occupied to the limit of his ability in merely keeping his sight on the target. Moreover, the firing of the individual guns must be slightly staggered in order that the gun pointer can identify the splashes from his particular gun; and with several guns firing, identification will still be difficult. Also the gun pointer must be carefully instructed as to the relation between the rate of fire of his gun and the times of flight for various approximate ranges, in order to avoid duplication of corrections on impacts of shots fired with the same deflection.

SECTION III

MAGNITUDE METHOD

■ 143. GENERAL.—In this method, the magnitude and the sense of the range deviation of the center of impact of a series of shots or salvos are the basis for determining the range correction to be applied. On the fire adjustment board, the impacts of all shots or salvo centers of impact are plotted in such a manner that each deviation is stripped of all adjustment correction. This allows the individual adjusting fire to base a correction on the center of impact of a series of shots, regardless of whether all of the shots of that series were fired on the same correction.

■ 144. TRIAL FIRE.—*a*. In the *ranging shot method* of trial fire, four shots are fired directly at the target, either by single shot or by salvo. Such shots are called ranging shots. In general, no correction as the result of observation of impacts is applied during trial fire. However, when time will permit and in cases where trial fire is by single shots or by two-gun salvos, if the deviation of the first shot or center of impact is large, a full correction based on this deviation may be made in order to bring the remainder of the trial shots closer to the target; the object being to permit a more accurate measurement of the deviations. In the latter case, if the first shot falls within three probable errors of the target, or if the center of impact of a two-gun salvo is within two gun probable errors of the target, no correction will be applied. The average of all the deviations, taking into consideration any corrections applied during the firing of the trial shots, furnishes the basis for the correction to be applied at the commencement of fire for effect.

b. In the *trial shot method at a fixed point*, which is used only for mortars, four shots are fired singly or in salvo, with the same data, at a fixed point in the water. The point chosen should be near an expected position of the target. The shots should be fired as rapidly as is consistent with accurate laying of the piece, preferably just before an engagement or practice. The correction to be applied before entering fire for effect is based on the mean deviation of the impacts from the trial shot point. As in the ranging shot method, if the trial fire is conducted by single shots or two-gun salvos and the deviation of the first shot or salvo center of impact is greater than three or two gun probable errors, respectively, a full correction based on this deviation may be applied. Usually four trial shots are fired in each zone in which action is anticipated.

■ 145. FIRE FOR EFFECT.—The correction determined as a result of trial fire is used to commence fire for effect. Four rounds are fired, and their center of impact combined with that of the four rounds of trial fire to form a basis for a correction. Thereafter, after each series of four rounds is fired, a correction is applied if necessary, such correction being based on the center of impact of the last eight shots. It will be noted that shots considered as a basis for a particular correction may or may not have been fired on the same correction. Following the general rules for fire adjustment, no correction of less than one-half of a gun probable error is applied. In case the fire for effect is conducted slowly and deliberately, corrections may be made at any time by taking the center of impact of the last eight shots. It is not necessary to wait for units of four impacts. The only reason for making corrections after each series of four shots is to eliminate confusion and large possibility of error in operating the fire adjustment board.

■ 146. EXAMPLES.—*a*. The following examples of range adjustment are based on the use of the fire adjustment board (See FM 4–15.) The standard system of reference numbers is used in which 300 represents a zero correction or a zero deviation, and the digit in the units' place represents tenths of 1 percent. For example, 315 represents ~~either a deviation of 1.5 percent over, or~~ a correction of up 1.5 percent. The data for the examples were determined by means of the dispersion tape and scale described in appendix I. A probable error of 1 percent is assumed for convenience in all examples.

b. In the examples, certain conventions have been followed as indicated below:

(1) A cross (×) is used to denote ~~the spotted deviation of~~ a single shot. (A cross with an exponent would be used to denote ~~the spotted deviation of~~ the center of impact of a salvo, the exponent being the number of shots in the salvo.)

(2) A small circle indicates the center of impact of a series of 4 shots.

(3) A check mark is used to show two things, the first being the location of the center of impact of the shots considered as a basis for a correction, and the second being the magnitude of the adjustment correction. ~~The magnitude of the correction is the distance from the axis of correction to the check mark or center of impact.~~

(4) The numbers immediately above a check mark indicate, in reference numbers, the correction ordered.

(5) The group of crosses used in determining a given center of impact is indicated by a bracket.

(6) The vertical scale is uniform, a horizontal line being used for each shot or salvo ~~in the effect phase~~. Ranging shots fired with the same or no adjustment correction are plotted on one line. No attempt has been made to plot against a vertical time scale, although this board readily lends itself to such a method. 2 PER LINE

(7) The symbol *S* indicates a shot or salvo, while *T. S.* indicates a trial shot or trial salvo.

c. For the examples in this section, the assumed situation is as follows:

Armament.	Two 8-inch guns (railway).
Range.	About 12,000 yards.
Firing interval.	40 seconds.
Time of flight.	30 seconds.
Time of spot.	5–8 seconds.
Data to guns before firing.	20 seconds.
Probable error.	1.0 percent.
Fire for effect.	2 gun salvos.

DOT - C.I. OF 2 X'S
0-2045 5-05-2 SALVOS

Example No. 1.—Tabulated data. POINT OF IMPACT

Shot No.	Uncorrected range plus ballistic correction (yards)	Adjustment correction		~~Range deviation~~		Corrected range (yards)
		Percent	Yards	Percent	Yards	
T. S. 1	12,050	300	0	~~342~~ 258	+510	12,050
T. S. 2	12,010	258	−500	280	−240	11,510
T. S. 3	11,970	258	−500	285	−180	11,470
T. S. 4	11,930	258	−500	296	−50	11,430
S-1	11,640	268	−370	280	−230	11,270
	11,640	268	−370	310	+120	11,270
S-2	11,530	268	−370	282	−210	11,160
	11,530	268	−370	286	−160	11,160
S-3	11,420	268	−370	303	+30	11,050
	11,420	268	−370	275	−290	11,050
S-4	11,300	273	−310	300	0	10,990
	11,300	273	−310	292	−90	10,990
S-5	11,180	273	−300	298	−20	10,880
	11,180	273	−300	306	+70	10,880
S-6	11,060	278	−240	293	−80	10,820
	11,060	278	−240	309	+100	10,820
S-7	10,940	278	−240	299	−10	10,700
	10,940	278	−240	305	+50	10,700

In example No. 1 (fig. 36), the first ranging shot was spotted at 342 or over 510 yards. Since this ~~deviation~~ SHOT was more than three probable errors from the target, ~~a correction~~ of 258 was ordered to bring the remaining trial shots closer to the target. The deviations of all four trial shots were considered in determining the correction of 268 with which fire for effect was opened. After the first four shots of fire for effect, the center of impact of these four shots was combined with that of the four ranging shots and a correction of one-half probable error, 0.5 percent, was indicated. It was considered that the state of training of the battery and the condition of the matériel warranted making this minimum allowable correction, and the correction was ordered. Before the correction took effect, however, the third salvo had been fired and the impacts of this salvo were therefore plotted on the same line of targets as the first and second salvos. The center of impact of salvos 3 and 4 was combined with that of salvos 1 and 2 and a further correction of 0.5 percent was

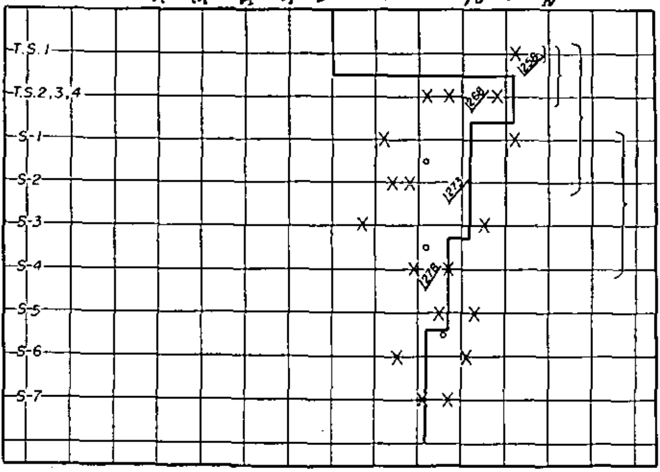


FIGURE 36.—Adjustment of fire, magnitude method (example No. 1).

applied, bringing the adjustment to 278. Salvo 5 was fired before the correction of 278 took effect and the impacts were plotted accordingly. The center of impact of salvos 5 and 6 was combined with that of salvos 3 and 4, but the combined center of impact was less than one-half probable error from the line of targets, and no correction was applied.

Example No. 2.—Tabulated data. POINT OF IMPACT

Shot No.	Uncorrected range plus ballistic correction (yards)	Adjustment correction		~~Range deviation~~		Corrected range (yards)
		Percent	Yards	Percent	Yards	
T. S. 1	10, 100	300	0	300	0	10, 100
T. S. 2	10, 400	300	0	324/~~[illegible]~~	−250	10, 400
T. S. 3	10, 730	300	0	316	+170	10, 730
T. S. 4	11, 030	300	0	270	−330	11, 030
S-1	11, 560	310	+120	284	−180	11, 680
	11, 560	310	+120	282	−210	11, 680
S-2	11, 760	310	+120	303	+40	11, 880
	11, 760	310	+120	286	−160	11, 880
S-3	11, 980	310	+120	295	−60	12, 100
	11, 980	310	+120	286	−170	12, 100
S-4	12, 180	310	+120	300	0	12, 300
	12, 180	310	+120	288	−150	12, 300
S-5	12, 390	310	+120	278	−270	12, 510
	12, 390	310	+120	286	−170	12, 510
S-6	12, 590	319	+240	292	−100	12, 830
	12, 590	319	+240	318	+230	12, 830

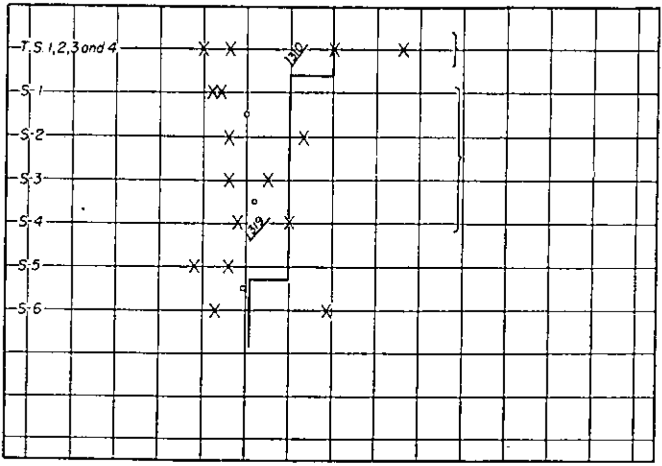


FIGURE 37.—Adjustment of fire, magnitude method (example No. 2).

In example No. 2 (fig. 37), the first ranging shot was spotted at 300 and the remaining shots of trial fire were fired with no adjustment correction. The center of impact of the four ranging shots indicated a correction of 310. Fire for effect was begun with this correction. The combined center of impact of the four trial shots and the first four rounds of fire for effect was just one-half probable error from the target. It was considered that the state of training of the battery did not warrant such a small correction, and no correction was ordered. However, the center of impact of the second series of four shots of record fire combined with that of the first series of four shots of record fire indicated a correction of 319. This correction was ordered, taking effect on salvo No. 6 of fire for effect. ~~This example of the magnitude method of range adjustment is given to illustrate the *corrections method of adjustment* briefly discussed in FM 4–15 in connection with the fire adjustment board. In this system, each impact or salvo center of impact is plotted on the adjustment board in terms of the additional correction that would have caused that individual impact to have fallen on the adjusting point; that is, the correction for each impact is plotted rather than the deviation of the impact. An impact, for example, whose range deviation in reference numbers is 314, is plotted as 286 in this system. The advantage of the corrections method is that the correction may be read directly from the chart at the center of impact of the shots considered, eliminating the possibility of a large personnel error in reading the scale at the wrong line on the chart.~~

PLOT WITH MOBILE SCALE (NUMBERED SAME AS BOARD) AND READ CORRECTION FROM BOARD.

Example No. 3.—Tabulated data. — CK - NEW METHOD

Shot No.	Uncorrected range plus ballistic correction (yards)	Adjustment correction		Point of impact		Corrected range (yards)
		Percent	Yards	Percent	Yards	
T. S. 1	13,050	300	0	312	−160	13,050
T. S. 2	12,900	300	0	331	−400	12,900
T. S. 3	12,800	300	0	320	−260	12,800
T. S. 4	12,700	300	0	328	−360	12,700
S-1	12,360	323	+280	303	−40	12,640
	12,360	323	+280	285	+190	12,640
S-2	12,280	323	+280	303	−40	12,560
	12,280	323	+280	314	−170	12,560
S-3	12,200	323	+280	320	−240	12,480
	12,200	323	+280	313	−160	12,480
S-4	12,120	323	+280	300	0	12,400
	12,120	323	+280	315	−180	12,400
S-5	12,020	323	+280	291	+110	12,300
	12,020	323	+280	307	−80	12,300
S-6	11,920	329	+350	302	−20	12,270
	11,920	329	+350	294	+70	12,270
S-7	11,820	329	+340	299	+10	12,160
	11,820	329	+340	315	−180	12,160

In this example (fig. 38) the first impact was within three probable errors of the target so trial fire was completed with no correction. The center of impact was at 323 and fire for effect was started with this correction. On the chart, the line of targets was drawn at the correction 323 and the impacts of fire for effect were plotted with this as a reference line as before. No correction as large as one-half probable error was indicated until after the eighth shot of fire for effect when a correction 329 was read directly from the chart over the center of impact of the 8 shots. This correction took effect on salvo No. 6. The center of impact of salvos 3, 4, 5, and 6 indicated that no further correction was required.

SECTION IV

BRACKETING METHOD

■ **147. GENERAL.**—In the bracketing method of range adjustment, only the sense of the impacts is spotted and the

GET BRACKET ... HANDPLE

correction is based on the relative number of overs and shorts. A hit should be counted as both an over and a short.

■ 148. TRIAL FIRE.—As a result of deductions in probability, certain rules for the application of corrections during and as a result of trial fire have been derived. These rules should be committed to memory. While the normal method is to fire by battery salvos, trial fire may be conducted by salvos

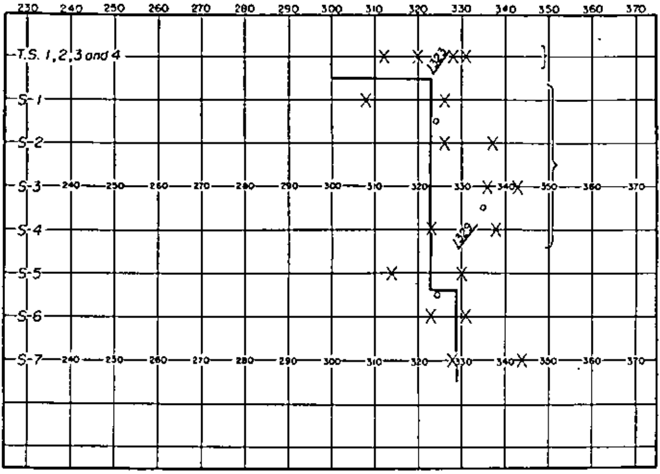


FIGURE 38.—Adjustment of fire, magnitude method (corrections) (example No. 3).

from two or more guns. Trial fire opens with the firing of one salvo. If the impacts of this salvo are sensed—

4-shot salvo	3-shot salvo	2-shot salvo
All in the same sense......	All in the same sense.....	All in the same sense.

an adjustment correction of one fork is applied and such correction repeated after each salvo until two corrections differing by one fork are determined, one of which gives overs and the other shorts. Fire for effect is started with that correc-

SEE ¶113
USE SALVO
SEE ¶116

tion which is the mean of the corrections giving the bracket. If, however, in attempting to obtain a bracket, any salvo gives a straddle, the correction with which to enter fire for effect should be obtained in the manner that applies, as indicated in *a* or *b* below.

a. If the impacts are sensed—

4-shot salvo	3-shot salvo	2-shot salvo
3 overs and 1 short or 3 shorts and 1 over. 1 hit and 3 overs or 1 hit and 3 shorts.	1 hit and 2 overs or 1 hit and 2 shorts.	

an adjustment correction of one probable error is applied in the proper direction. This is the correction with which to enter fire for effect.

b. If the impacts are sensed—

4-shot salvo	3-shot salvo	2-shot salvo
2 overs and 2 shorts. 1 hit as well as 1 or more overs and 1 or more shorts. 2 or more hits.	1 hit, 1 over, and 1 short. 2 or more hits. 2 overs and 1 short or 2 shorts and 1 over.	1 over and 1 short. 1 or more hits.

no change is made; the correction with which this salvo was fired is the correction with which to enter fire for effect.

■ 149. Fire for Effect.—*a*. Corrections during this phase are determined by the formula

$$\frac{\text{Overs}-\text{shorts (or shorts}-\text{overs)}}{2\times(\text{overs}+\text{shorts})}\times 1 \text{ fork}$$

in which a hit is counted as both an over and a short. To facilitate the determination of corrections from the formula, the bracketing adjustment chart is constructed. (See par. 150.)

b. Fire for effect is carried out as continuous fire, there being no delay for the application of corrections. In general at least eight and not more than twelve shots should be considered in determining adjustment corrections to be applied during this phase. However, if fire for effect is started with a correction which is the mean of the corrections giving a bracket (par. 148), and *if the first four or more impacts of fire for effect* are all of the same sense, a correction of one-half fork should be applied immediately.

c. In determining corrections to be applied during fire for effect, the last 8 to 12 shots fired with the same adjustment should be considered, even though some of these shots were fired during trial fire.

■ 150. EXAMPLES.—The following examples of range adjustment problems are based on the use of the grid type bracketing adjustment chart described in FM 4–15. The standard system of reference numbers is used in which 300 represents a zero correction and the digit in the unit's place represents tenths of 1 percent. For example, 313 represents an up correction of 1.3 percent. The symbol T.S. indicates "trial salvo," and in the illustrations of the adjustment chart a small circle around a numeral on the chart indicates that at that point in the firing an adjustment correction was ordered and its magnitude in tenths of 1 percent is indicated by the circled numeral. The assumed situation for the examples is as follows:

Armament.	155-mm. guns.
Range.	About 11,000 yards.
Firing interval.	20 seconds.
Time of flight.	30 seconds.
Data to guns before firing.	20 seconds.
Fork.	2.8 percent. - MIN COR, 3.5
Method of trial fire.	2-gun salvo.
Fire for effect.	4-gun salvo.

NO C.I OF SALVO - BUT SHOT BY SHOT

a. Example No. 1.—A battery of four 155-mm guns using normal charge was fired at a target according to the following tabulation:

Shot No.	Range sensings	Adjustment correction		Uncorrected range plus ballistic correction (yards)	Corrected range (yards)
		Percent	Yards		
T. S. 1	O-O	300	0	10, 100	10, 100
T. S. 2	S-S	272	−290	10, 200	9, 910
1	S-H-S-S	286	−150	10, 300	10, 150
2	O-S-O-S	286	−150	10, 400	10, 250
3	H-S-S-O	286	−150	10, 510	10, 360
4	O-S-S-S	286	−150	10, 620	10, 470
5	H-O-S-O	291	−100	10, 720	10, 620
6	S-S-H-O	291	−100	10, 820	10, 720
7	O-H-O-O	291	−100	10, 930	10, 830

The procedure of adjustment in this problem is illustrated in figure 39. The first trial salvo was over and a correction of down one fork (−2.8 percent) was applied. The second trial salvo fired on the new adjustment was short. This gave two corrections, separated by one fork, one of which gave

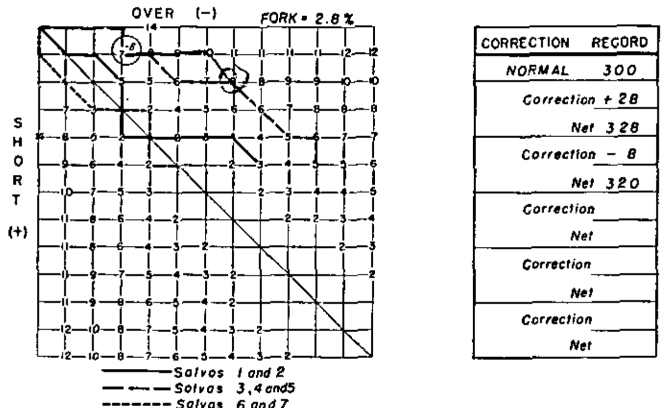


FIGURE 39.—Adjustment of fire, bracketing method (example No. 3).

overs and the other shorts. The mean of the two corrections was then used to commence fire for effect. This correction was 286. After two salvos of four shots each, a correction of up 0.5 percent was indicated on the adjustment chart and the correction was ordered. However, salvos 3 and 4 had been fired on the original adjustment before the correction could take effect. These two salvos were plotted on the chart on the same line as the impacts from salvos 1 and 2. The correction took effect on salvo 5 and with this group of impacts a new line was started on the chart. The impacts of salvos 5, 6, and 7 indicate that an effective adjustment of fire has been accomplished, because after the eighth round of that series no correction as large as one-half a probable error was called for.

b. Example No. 2.—The battery in *a* above fired at another target as follows:

Shot No.	Range sensings	Adjustment correction		Uncorrected range plus ballistic correction (yards)	Corrected range (yards)
		Percent	Yards		
T. S. 1	O-O	300	0	11,880	11,880
T. S. 2	S-S	272	−330	11,780	11,450
1	O-O-O-O	286	−170	11,680	11,510
2	O-S-O-O	286	−170	11,570	11,400
3	H-O-O-O	286	−160	11,470	11,310
4	S-H-S-S	272	−320	11,370	11,050
5	O-O-H-S	272	−310	11,260	10,950
6	S-S-O-S	272	−310	11,160	10,850
7	O-S-H-S	272	−310	11,070	10,760

Figure 40 illustrates the adjustment of fire in this example. The first trial salvo was over and a correction of down one fork or 2.8 percent was ordered. The second trial salvo was short, establishing a bracket. A correction of up one-half fork was applied and fire for effect was begun. Salvo No. 1 produced four impacts, all over. A correction of down one-half fork, 1.4 percent, was applied immediately. (This correction followed the rule which states that if fire for effect is begun with the mean of two corrections which gave a bracket in trial fire, and the first four impacts occur all in the same sense, a correction of one-half fork should be made.) Salvos Nos. 2 and 3 were fired on the opening adjustment and were plotted on the chart with salvo No. 1. The correction took effect on salvo No. 4, with which a new line of impacts was started. After salvo No. 6, a correction of up 0.4 percent

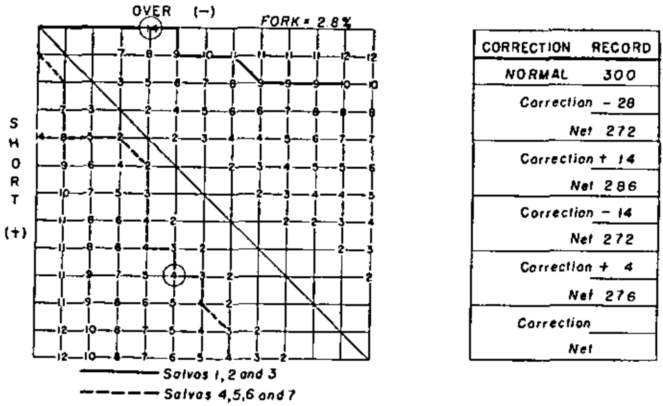


FIGURE 40.—Adjustment of fire, bracketing method (example No. 2).

was indicated, and this being slightly more than one-half probable error, it was applied.

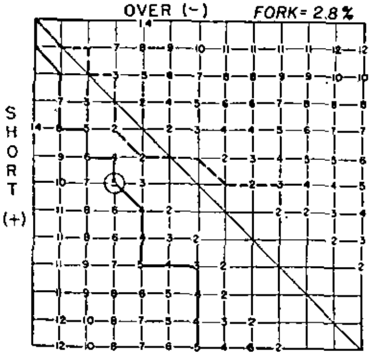
c. Example No. 3.—The following tabulation shows the data of a firing by the same battery of 155-mm. guns.

Shot No.	Range sensings	Adjustment correction		Uncorrected range plus ballistic correction (yards)	Corrected range (yards)
		Percent	Yards		
T. S. 1	SS	300	0	10,020	10,020
T. S. 2	HO	328	+280	10,040	10,320
1	H-S-S-O	328	+280	10,070	10,350
2	O-O-O-H	328	+280	10,090	10,370
3	O-O-O-S	328	+280	10,120	10,400
4	O-H-O-O	328	+280	10,150	10,430
5	H-H-O-S	328	+280	10,190	10,470
6	S-H-H-O	320	+200	10,220	10,420
7	O-S-S-O	320	+200	10,250	10,450

4-GUN TRIAL SALVES

39

Figure ~~41~~ shows the adjustment of fire in this practice. The first trial salvo was short, causing a correction of up one fork, or 2.8 percent, to be applied. The second trial salvo produced a hit and an over and fire for effect was begun with this same adjustment. The impacts of the second trial salvo were plotted on the chart to be considered with the first impacts of fire for effect. After salvos Nos. 1 and 2 of fire for effect had been plotted, no correction was found necessary. However, at this time it was noted that the center of impact of salvo No. 2 was definitely over, and it was decided to observe



CORRECTION RECORD	
NORMAL	300
Correction	− 28
Net	272
Correction	+ 14
Net	286
Correction	+ 5
Net	291
Correction	
Net	
Correction	
Net	

FIGURE 41.—Adjustment of fire, bracketing method (example ~~No. 6~~). No. 1

the next salvo carefully to see if its center of impact also was over. (A new line of impacts was started on the chart with salvo 3 because a correction should not be based on more than 12 impacts and one more salvo on the first line of impacts would have made 14 impacts.) Salvo No. 3 was spotted as three overs and a short, confirming the suspicion that a down correction was in order. In making a correction at this point in the fire for effect phase, it was necessary to consider at least 8 shots. Therefore, salvo No. 2 was used by *mentally* placing the four impacts of that salvo on the new line of impacts after those of salvo No. 3. The indicated correction obtained in this manner was —0.8 percent. This was applied.

The indicated correction was written in the circle at the place on the chart where the correction was ordered, and further shots that were fired on the old adjustment were plotted as usual.

SECTION V

CONSIDERATIONS AFFECTING ADJUSTMENT OF FIRE

■ 151. WARMING-UP EFFECT.—It sometimes appears that there is a marked difference in range between the first and succeeding shots from a cold gun. The records of previous firings should be studied to determine whether or not such a warming-up effect has been shown by the guns. In most cases, it will be found that no such effect is apparent; however, where it has usually made its appearance in the past, adjustment of fire should be planned in such a way as to take account of it in the future. The simplest procedure would be to disregard the first shot in determining the correction with which to begin fire for effect.

■ 152. TACTICAL SITUATION.—The requirements of the tactical situation will determine the rate of fire and will determine whether fire should be interrupted for the purpose of applying corrections. If the tactical situation requires a rapid rate of fire, it is obvious that all guns will be used, firing should be as rapid as possible, and corrections should be applied without interrupting fire. There will be no trial fire as such. However, it will be unusual when time is not available for the firing of at least one ranging salvo and for the suspension of fire for the application of adjustment corrections based on this salvo. When fire is slow, it appears reasonable that trial fire should be conducted deliberately, and that necessary time out should be taken to apply trial fire corrections. To carry this principle further, when fire is so slow and deliberate that it may be conducted by single shots, a correction may be applied after each round is spotted (based progressively on the center of impact), placing the center of dispersion ever nearer the target as its location becomes more definite.

CHAPTER 12

GLOSSARY OF TERMS

■ 153. GLOSSARY.

Absolute deviation.—Shortest distance from center of the target to point of impact.

Accuracy of fire.—Accuracy of fire is determined by dispersion and is measured by the closeness of the grouping of points of impact about their center of impact.

Accuracy of practice.—Accuracy of practice is measured by the distance of the center of impact from center of target. It is sometimes known as accuracy of the shoot.

Adjusted range correction.—That range correction obtained or proved by actual firing which places the center of impact at or near the target.

Adjusting point.—The particular part of the objective on which fire is adjusted.

Aerial observation.—Observation of fire from balloons, airplanes, or airships.

Aiming point.—Point on which the gun pointer sights in pointing the gun in direction.

Altitude.—Vertical distance above or below a specified datum level, usually sea level at mean low water. It is sometimes called *height of site.*

Angle of depression.—Angular depression of the line of site below the horizontal plane.

Axial observation.—Observation of fire from a point on or near the gun—target line. An axial station is one from which the angle gun—target—station is less than 5°.

Axis of the bore.—Center line of bore of the gun.

Axis of trunnions.—Axis about which a cannon is rotated in elevation.

Azimuth.—Horizontal angle, measured in a clockwise direction, from a reference line passing through the position of the observer, to the line joining the observer and the objective. For seacoast artillery, the reference line is parallel to the true south line through the origin of the local system of rectangular coordinates.

Azimuth difference.—The difference, due to displacement, between the two azimuths of a point as measured from two other points. It is also called *parallax.*

Ballistic density.—A fictitious constant density of the atmosphere which would have the same total effect on the projectile during its flight as the varying densities actually encountered.

Ballistic wind.—A fictitious wind, constant in magnitude and direction, which would have the same total effect on the projectile during its flight as the true winds actually encountered.

Base line.—A line of known length and direction between two observation stations, the positions of which with respect to the battery are known.

Bilateral observation.—Observation of fire from two observation stations.

Bore rest.—See *clinometer rest.*

Bore sighting.—Process by which the axis of the bore and the line of sight are made parallel or are made to converge on a point.

Bracketing correction.—An adjustment correction which gives an equal number of overs and shorts.

Bracketing salvo.—A salvo in which the number of impacts sensed short is equal to the number of impacts sensed over.

Calibration.—Determination, by actual firing, of elevation and deflection corrections to be applied to the individual pieces of the battery in order that their true centers of impact will be brought as close together as possible.

Case I pointing
Case II pointing } See *pointing.*
Case III pointing

Center of dispersion.—See *dispersion* (par. 101).

Center of impact.—Point whose deviation is the mean of the deviations of the several shots of a series.

Chronograph.—An instrument for measuring and recording short intervals of time. More specifically, an instrument for determining the velocity of projectiles.

Clinometer.—An instrument used on a clinometer rest to measure the inclination of the axis of the bore to the horizontal.

Clinometer rest.—A device inserted in the bore of a cannon at the muzzle for supporting the clinometer. It is also called a *bore rest*.

Coefficient of form.—A factor introduced into the ballistic coefficient to make its value conform to results determined by firing.

Conduct of fire.—Employment of technical means to place accurate fire on a target. Fire is usually conducted by the battery, which is the normal fire unit.

Continuous fire.—Fire conducted at the normal rate without interruption for the application of adjustment corrections or for other causes.

Corrected azimuth.—Azimuth from directing point to the target corrected for all known variations from those conditions assumed as standard in the construction of firing tables.

Corrected deflection.—Deflection corrected for all known variations from those conditions assumed as standard in the construction of firing tables.

Corrected elevation.—Firing table elevation corresponding to the corrected range.

Corrected range.—Range from directing point to the target corrected for all known variations from those conditions assumed as standard in the construction of firing tables.

Danger space.—Area indicated by projecting the target onto the surface of the water by lines parallel to line of fall of the projectile.

Datum level.—A spherical surface which represents mean sea level or other established reference level from which altitudes are measured.

Datum point.—A fixed point whose ranges and azimuths from fixed elements of the defenses have been accurately determined.

Dead time.—Time elapsed between the instant an observation is taken on the target and the instant the cannon is fired with firing data that were calculated as a result of that observation.

Defilade.—Vertical distance by which a position is concealed from enemy observation. If the smoke and flash of firing are also concealed, the battery is said to have smoke and flash defilade.

Deflection.—Horizontal angle between line of sight to the target and axis of the bore when the piece is pointed in direction. It is usually expressed in reference numbers and is set on the sight. Deflection due to travel alone is called *uncorrected deflection.*

Deliberate fire.—Fire which is conducted at a rate intentionally less than the normal rate of fire of the battery for the purpose of applying adjustment corrections between series or for tactical reasons.

Deviation.—See paragraph 102*a*.

Difference chart.—A graphic device by means of which the range and azimuth of a target from a gun or station are obtained when the range and azimuth from some other gun or station are known.

Directing point.—A point in or near a battery for which the range and azimuth to the target are determined in computing firing data. If a gun of the battery is the directing point, it is called the *directing gun.*

Directrix.—Center line of field of fire of a gun or battery.

Dispersion ladder.—A diagram made up of eight successive zones, each equal to one probable error, in each of which is indicated the percentage of shots expected to fall therein; the center of dispersion is on the line between the two central zones. There is a dispersion ladder for range and one for direction. (See par. 101*d*.)

Displacement.—Horizontal distance in yards from one point to another.

Drop.—Vertical distance from a point on the trajectory to line of elevation.

Elevation table.—A table of ranges with corresponding quadrant elevations used in graduating or checking graduations of the range disk of a fixed gun. The quadrant elevations listed are firing-table elevations corrected for height of site.

Field of fire.—The portion of the terrain or water area covered by fire of a gun or battery.

Fifty percent zone.—Zone extending one probable error on each side of the center of dispersion within which 50 percent of the shots are expected to fall.

Fire control.—Exercise of fire direction and conduct of fire. Fire-control equipment and installations are used both for the tactical direction of fire and for the technical conduct of fire.

Fire direction.—Exercise of the tactical command of one or more units in the selection of objectives and in the concentration or distribution of fire thereon at appropriate times.

Fire for effect.—Fire which has for its primary object the accomplishment of tactical effect sought.

Firing azimuth.—Corrected azimuth further corrected for an individual cannon. It includes individual corrections for displacement and calibration.

Firing data.—A general term employed in speaking of the range (or elevation) and azimuth (or deflection), either corrected or uncorrected, that are used in pointing a cannon.

Firing elevation.—Firing table elevation corresponding to firing range.

Firing range.—Corrected range further corrected for an individual cannon. It includes individual corrections for displacement, lack of level of base ring, and calibration.

Firing tables.—Collection of data, chiefly in tabular form, intended to furnish the ballistic information necessary for conducting the fire of a particular model of gun and mount with specified ammunition.

Flank observation.—Observation of fire from a point on or near the flank. A flank station is one from which the angle battery—target—station is greater than 75°.

Flash defilade.—See *defilade.*

Fork.—Difference in range or elevation or in direction required to change the center of impact by four probable errors.

Grid azimuth.—Azimuth measured from grid north. Formerly called the *Y* azimuth.

Gun difference.—Difference, due to displacement, between the range from a gun to the target and range from directing point to the target.

Gun displacement.—Horizontal distance from the pintle center of the gun to the directing point or directing gun of a battery.

Gunner's quadrant.—An instrument used on a quadrant seat on the breech of a gun to measure the inclination of the axis of the bore to the horizontal.

High angle fire.—Fire delivered at elevations greater than the elevation corresponding to the maximum range.

Hit.—An impact on the target.

Hitting area.—Arbitrarily defined as the area extending three probable errors on each side of the center of dispersion, in range as well as in direction.

Horizontal base system.—A system of position finding in which the target is located by the intersection of two lines of known direction from two observing stations.

Hundred percent rectangle.—A rectangle whose length is eight probable errors in range and whose breadth is eight probable errors in direction. Its center is the center of dispersion. It is expected to contain practically all of the shots.

Jump.—Angle between the line of elevation and line of departure. Its component in a vertical plane is called *vertical jump* and its component in a horizontal plane is called *lateral jump.*

Lateral deviation.—See *deviation.*

Lateral jump.—See *jump.*

Level point.—Point on descending branch of the trajectory at the same altitude as the muzzle of the gun. Same as *point of fall.*

Line of collimation.—Line from the center of the objective lens of a telescope through and perpendicular to the axis of vertical rotation.

Line of departure.—Prolongation of axis of bore as the projectile leaves the muzzle of the gun. It is tangent to the trajectory at the origin.

Line of elevation.—Prolongation of axis of the bore when the piece is laid.

Line of fall.—The tangent to the trajectory at the level point.

Line of impact.—The tangent to the trajectory at point of impact.

Line of position.—Same as *line of site.*

Line of site.—Line of site of a point is the straight line connecting the origin of the trajectory with that point. Also called *line of position.*

Longitudinal deviation.—See *deviation.*

Low angle fire.—Fire delivered at angles of elevation below that required for maximum range.

Magnitude method of adjustment.—A method of adjustment used when deviations are measured by spotting.

Map range.—Range from the piece to any point as scaled or computed from a map.

Mask.—Any natural or artificial feature of or on the terrain which affords shelter from view.

Maximum ordinate.—Difference in altitude between the gun and the summit of the trajectory.

Meteorological datum plane.—Datum level containing the meteorological station from which meteorological conditions are measured.

Mil.—One sixty-four-hundredth part of a circle. For practical purposes, the arc which subtends a mil at the center of a circle is equal in length to 1/1000 of the radius. The arc and its tangent are nearly equal for angles not greater than 330 mils.

Mistakes.—Personnel errors which may be avoided by proper care.

Normal of a scale.—Reference number that represents zero units of the value concerned.

Objective plane.—Plane tangent to the surface of the target at point of impact.

Observing line.—Line joining the observer and observing point.

Observing point.—Point on which the observer sights.

Observing sector.—Sector between the lines to the right and left limiting the area visible to the observer, or limiting the area assigned for surveillance.

Orientation.—*a.* Determination of the horizontal and vertical location of points and the establishment of orienting lines, or lines of known direction.

b. Adjustment of an instrument or gun to read azimuths.

Orienting line.—A line of known direction, over one point of which it is possible to place an angle-measuring instrument.

Parallax.—Angle subtended at a certain point by a line connecting two other points; also called azimuth difference.

Pattern.—The pattern of a salvo in range is the difference in range between the point of impact with the longest range and the point of impact with the shortest range, excluding wild shots. The pattern of a salvo in direction is the distance measured perpendicular to the line of position between the point of impact falling at the greatest distance to the right, and that falling at the greatest distance to the left, excluding wild shots.

Pintle center.—Vertical axis about which a gun and its carriage are traversed.

Plane of departure.—Vertical plane containing the line of departure.

Plane of fire.—Vertical plane containing the axis of the bore when the piece is laid.

Plane of site.—A plane containing the line of site and a horizontal line perpendicular to it.

Plotted point.—A point on the plotted course of the target located by means of observations taken at the end of an observing interval.

Pointing.—Operation of giving the piece a designated elevation and direction. There are three general cases of pointing:

Case I.—Pointing in which both direction and elevation are given to the piece by means of a sight pointed at the target.

Case II.—Pointing in which direction is given to the piece by means of a sight pointed at the target, and elevation by means of an elevation quadrant or a range disk.

Case III.—Pointing in which direction is given to the piece by the azimuth circle or by a sight pointed at an aiming point other than the target, and elevation by means of an elevation quadrant or a range disk.

Position finding.—Process of determining the range and direction of a target.

Predicted point.—A point at which it is expected the target will arrive at the end of the dead time interval.

Predicting.—Process of determining the expected position of the target at some future time.

Predicting interval.—~~A specified interval of time allowed for the processes of calculating the firing data and for lay-~~

INTERVAL BETWEEN SUCCESSIVE

PREDICTIONS OF S.F.P.'S

~~ing the guns. The length of the predicting interval varies for different fire control systems.~~

Probability factor.—A factor used as an argument in entering the probability tables. It is equal to the error not to be exceeded divided by the probable error.

Quadrant angle of elevation.—Angle between the line of elevation and horizontal plane at the muzzle. (See par. 9*j*.)

Quadrant angle of fall.—Angle between the line of fall and horizontal plane at the level point. (See par. 10*h*.)

Quadrant angle of site.—Angle between the line of site and horizontal plane at the muzzle.

Range.—Horizontal distance from the gun, observing station, or directing point of a battery to the target, splash, datum point, or other specified point.

Ranging shots.—Trial shots fired at a moving target for the purpose of obtaining an adjustment correction to be used in entering fire for effect.

Reference line.—A line to which directions or azimuths are referred. A line of zero azimuth for a particular system would be a reference line.

Reference numbers.—Arbitrary numbers used in place of actual values in the graduation of certain scales of instruments used in gunnery. Their purpose is to avoid the use of positive and negative values.

Relocation.—Process of determining without further observations the range and azimuth from one station to a point (or target) when the range and azimuth from another station to this point are known.

Remaining velocity.—Remaining velocity at any point in the trajectory is the actual velocity in feet per second at that point.

Retardation.—In ballistics, the negative acceleration of the projectile.

Ricochet.—A glancing rebound of a projectile.

Round.—All of the component parts of ammunition necessary in the firing of one shot.

Salvo.—One shot per gun, fired simultaneously or fired in a certain order with a specified time interval between rounds.

Salvo point.—A point of known range and azimuth at which fire from one or more batteries may be directed.

Self-contained range finder.—An instrument used to obtain ranges by either the stereoscopic or the coincidence principle.

Sense.—Direction of a point of impact (or center of impact of a salvo) with respect to the target, as "over," "short," "right," or "left."

Set-forward point.—A point on the expected course of the target at which it is predicted the target will arrive at the end of the predicting interval plus the time of flight.

Slope of fall.—Degree of inclination of line of fall to the horizontal. It is usually expressed as a gradient, for example one on five, meaning that the projectile drops vertically 1 yard while it is moving horizontally through 5 yards.

Smoke defilade.—See *defilade.*

Spotting.—Process of determining deviations or sensings for use in the adjustment of fire.

Straddle.—A salvo which has impacts of opposite sense. Also called a *mixed salvo.*

Striking velocity.—Remaining velocity at the point of impact.

Stripped deviation.—Deviation that would have resulted had there been no personnel errors and no adjustment corrections applied.

Subareas.—Subdivisions of the water area in the field of fire used to assist in the indication and identification of targets.

Summit of trajectory.—Highest point on the trajectory.

Terminal velocity.—Remaining velocity at the point of fall.

Time of flight.—Elapsed time from instant of departure of the projectile to instant of impact.

Tracking.—Process of making successive observations on a moving target for the purpose of plotting its course.

Travel of the projectile.—Distance from the base of the projectile in its seat to the face of the muzzle of the gun.

Trial shot correction.—Adjustment correction resulting from trial fire and used in entering fire for effect.

Trial shot point.—A point in the field of fire, visible from the observing stations, at which trial shots are fired.

Twenty-five percent rectangle.—That portion of the dispersion diagram, the dimensions of which are two probable errors in range by two probable errors in direction, and the center of which is on the center of dispersion. It is that area which is common to the 50 percent zones in range and direction and within which 25 percent of the shots are expected to fall.

Uncorrected deflection.—The deflection due to travel of the target during time of flight.

Unilateral observation.—Observation of fire from a point so located that the angle battery—target—station is between 5° and 75°.

Vertical base system.—A system of position finding in which the target is located by direction and distance from a single station using a depression position finder.

Vertical jump.—See *jump.*

Wild shot.—A shot whose impact is more than four developed armament probable errors or, in the absence of this information, more than four firing table probable errors from the center of impact.

Yaw.—The angle between the longitudinal axis of the projectile and the tangent to the trajectory at center of gravity of the projectile.

Zone.—The area between two limiting ranges in which projectiles will fall when one particular size powder charge is used and the elevation is varied from the minimum to the maximum.

Zones of dispersion.—See *dispersion* (par. 101).

APPENDIX I

DISPERSION TAPE AND SCALE FOR USE IN FIRE ADJUSTMENT PROBLEMS WITH SIMULATED FIRE

■ 1. GENERAL.—Opportunities to practice the principles of adjustment of fire during actual firing are comparatively rare. If an individual is to become proficient in this art, he should be given the opportunity to practice the use of the rules involved by means of simulated fire. The value of such practice is enhanced if it is given under conditions which are similar to actual firing conditions. The firing itself will of necessity be simulated. The dispersion tape and scale described herein afford the means of simulating the results of fire and determining the effect of adjustment corrections on the position of the dispersion zone with reference to the target.

■ 2. SIMULATED DISPERSION.—*a.* In order to practice adjustment of fire during simulated fire, it is necessary to provide a means of obtaining the dispersion that occurs in actual firing. Gun dispersion follows the law of accidental errors. Therefore, it is first necessary to devise some method to produce a normal distribution of errors.

b. For the dispersion tape, the simulated dispersion is obtained by means of two dice of different colors. On throwing these dice simultaneously any one of 36 possible combinations is likely to occur. By reading the same color first each time, each of these combinations is represented by a different number consisting of two digits. Therefore, if the area under a typical curve of errors be divided equally into 36 vertical strips and each of the 36 dice combinations is assigned to a particular strip as shown in figure 1, a method is evolved of simulating dispersion of gun fire. It will be noted that since the strips are of equal area, they are of decreasing width toward the center of dispersion; and like the fall of shots from a gun, they will be grouped densely near the center and sparsely toward the ends of the dispersion curve. Each strip represents the probability of throwing a particular combination of the two dice and its distance from the vertical center line of the curve may be considered to be the deviation of a shot from the center of dispersion. (The distance is measured between the vertical center line of the curve and a

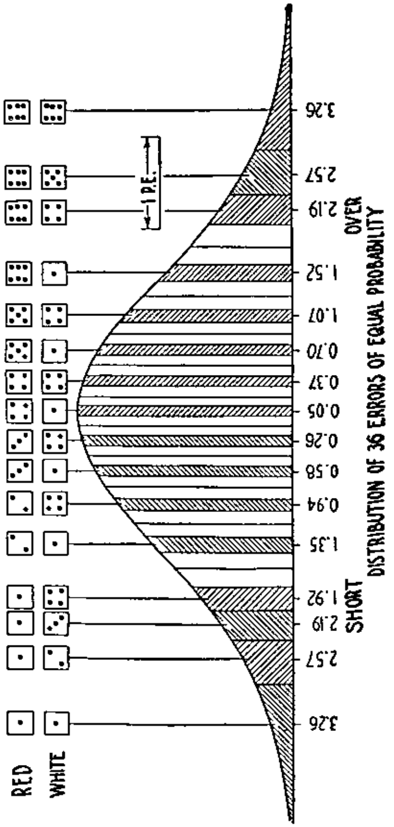


FIGURE 1.—Normal distribution, two dice.

vertical line bisecting the area of the strip.) The distribution table accompanying figure 1 gives the data for the arrangement shown in the figure and is used in constructing the dispersion tape described in paragraph 3.

DISTRIBUTION TABLE

Spots on—		Armament error indicated in PE's	Spots on—		Armament error indicated in PE's
Red	White		Red	White	
1	1	−3.26	4	1	+0.05
1	2	−2.57	4	2	+.15
1	3	−2.19	4	3	+.26
1	4	−1.92	4	4	+.37
1	5	−1.71	4	5	+.47
1	6	−1.52	4	6	+.58
2	1	−1.35	5	1	+.70
2	2	−1.20	5	2	+.81
2	3	−1.07	5	3	+.94
2	4	−.94	5	4	+1.07
2	5	−.81	5	5	+1.20
2	6	−.70	5	6	+1.35
3	1	−.58	6	1	+1.52
3	2	−.47	6	2	+1.71
3	3	−.37	6	3	+1.92
3	4	−.26	6	4	+2.19
3	5	−.15	6	5	+2.57
3	6	−.05	6	6	+3.26

■ 3. DISPERSION TAPE.—The dispersion tape is a roll of reinforced paper divided into several hundred frames or sections on each of which are placed four vertical marks to represent splashes. The center line of the tape represents the center of dispersion and the marks are positioned in each frame according to the fall of the two dice and the table in paragraph 2*b* above. For proper identification of shots in simulated salvo firing, the marks bear different symbols. One bears a cross (X), another a circle (O), a third a double bar (=), and a fourth is left plain. The tape is mounted on rollers and is covered so that only one frame appears at a time under a window in the cover. (Dispersion tapes and plans for constructing the mount may be obtained, upon request, from the Coast Artillery Board.)

■ 4. DEVIATION SCALE (fig. 2).—*a. General.*—(1) This is a movable scale placed in a guide under the window of the dispersion tape mount and is graduated with the standard

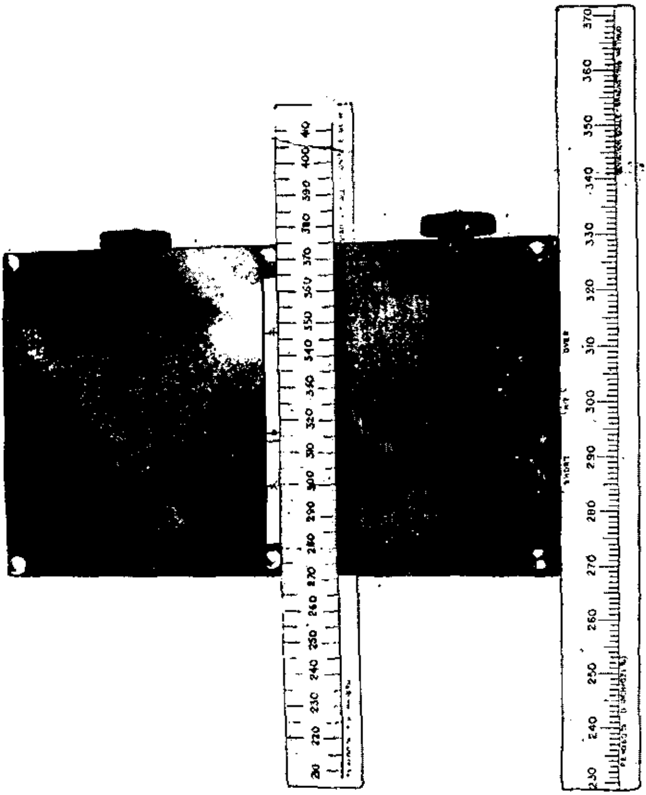


FIGURE 2.—Dispersion tape and scale.

range reference numbers representing percentages of range The graduations increase from left to right, with the midpoint, marked 300, representing the position of the target The scale of the graduations should be such as to fit the dispersion zone of the dispersion tape and the probable error of the armament manned. The depth of the dispersion zone on the tape issued by the Coast Artillery Board is 6.8 inches representing 8 P. E. The scale of graduations may be determined from this relationship. For example, if the probable error in range for the armament is 0.6 percent, the scale of the graduations should be

$$1 \text{ percent} = \frac{6.8}{8 \times 0.6} = 1.42 \text{ inches}$$

(2) The deviation scale is also marked at the upper edge to show "overs," "shorts," and "hits" for use with the bracketing method of adjustment. The width of the space marked "hit" on the deviation scale may be determined from the size of the danger space of the average target at medium range.

b. Instructions for use.—(1) The operator determines deviations by reading from the scale opposite the marks on the tape. He reads as many deviations from a frame as are needed for the salvo and moves the tape a predetermined number of frames to get the next set of deviations. An exceedingly great number of combinations of deviations may be secured by changing the selection of marks to be read or the number of frames to be turned between readings. The tape may be turned in either direction. The only precaution necessary is that the selection of frames and marks be made by some predetermined rule which should be followed until the end of the problem in order to insure that the dispersion depends on chance.

(2) Proper simulation of timing is one of the most important, and at the same time difficult, elements of successful drill. Except for such preliminary instructions as may be necessary, all problems should be conducted with the same timing as would be required during the firing of an actual practice. No deviation should be reported to the person conducting the adjustment until the end of the period required for the time of flight and the normal functioning of the

spotting section. The latter operation may be assumed to take from 5 to 10 seconds. Corrections should not be applied to the firing data sooner than could be done normally.

(3) Care should be taken to keep the position of the deviation scale in step with the firing data. An adjustment correction should not be applied on the dispersion device until the fall of the shot fired with that correction.

c. Operation in drill.—(1) Place deviation scale in position under window, move normal (300) graduation the desired distance from the center of dispersion, and mark this position with a pin placed on the mount.

(2) Determine a rule to be followed in selecting deviations and, following that rule, bring the proper frame on the dispersion tape into view in the window.

(3) At the proper time, read the deviation from the deviation scale opposite the mark that represents the splash.

(4) If an adjustment correction is ordered, move the deviation scale until the correction ordered is opposite the pin, timing the move to synchronize with the fall of the shot on which the new correction is applied.

(5) Do not move the pin that marks the initial position of the normal (300) graduation until the problem is completed unless it is desired to simulate a shifting center of dispersion. If such action is desired, shift the pin and deviation scale together in the amount and direction desired.

■ 5. CHANGES REQUIRED WHEN USING CORRECTIONS METHOD OF FIRE ADJUSTMENT.—*a.* When using the corrections method of fire adjustment, mentioned as an alternate method in FM 4–15, the dispersion slide rule is augmented by an additional scale pinned or fixed on the mount just below the deviation scale. This additional scale, which for convenience will be called the fixed scale, is a duplicate of the deviation scale and is fastened with the normal (300) opposite the center of the dispersion tape.

b. The operation of the dispersion device is exactly as described in paragraph 4c except that the pin is placed on the deviation scale at the normal of the fixed scale, and when an adjustment correction is ordered, the deviation scale is moved until the pin is opposite the correction ordered on the fixed scale.

APPENDIX II

ALINEMENT DIAGRAM GIVING PROBABILITY OF HITTING

INSTRUCTIONS FOR USE

■ 1. The alinement diagram (fig. 3) performs the operations described in paragraph 127. It divides the length of the danger space by the length of the 50-percent zone, or one-half the danger space by the probable error, and shows at once the probability corresponding to the resulting factor.

■ 2. Scales A and B may be considered as graduated in yards, tens of yards, hundreds of yards, or any other units desired, but both must be considered as graduated the same.

■ 3. When the center of dispersion is on the center of the danger space, hold a straightedge so that it intersects scale A at the graduation corresponding to the length of the danger space and scale B at that corresponding to the length of the 50-percent zone. Where the straightedge intersects scale C, read the value of the probability. Or on scale A, set one-half the length of the danger space and on scale B the probable error, reading as before on scale C.

■ 4. Scale D is graduated in values one-half of those on scale C. It is used when the danger space is not symmetrical with respect to the center of dispersion. If the center of dispersion is at the edge of the danger space, set the length of the danger space on scale A and the probable error on scale B. Read the corresponding probability on scale D.

■ 5. If the center of dispersion is within the danger space but not at its center, use the diagram twice. First, find the probability of hitting that part of the danger space which lies beyond the center of dispersion, using scales A, B, and D. Second, find the probability of hitting that part of the danger space which lies short of the center of dispersion, using again scales A, B, and D. The sum of these two results is the probability of hitting.

■ 6. If the center of dispersion does not lie in the danger space, first find the probability of hitting between the center of dispersion and the farther edge of the danger space and second, the probability of hitting between the center of dispersion and the nearer edge. Use scales A, B, and D for both

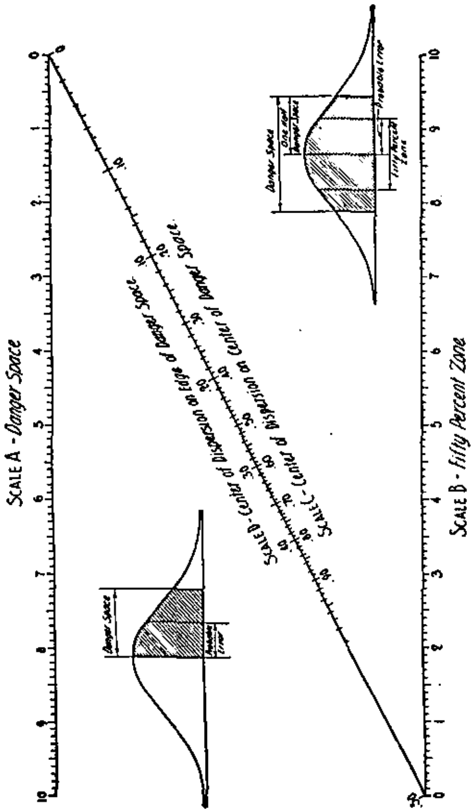


FIGURE 3.—Alinement diagram giving probability of hitting.

operations. The difference between these two results is the required probability of hitting.

■ 7. If the value set on scales A and B gives poor intersection on scale C or D, multiply both values by some number that will move them farther from the zeros of the scales. For example, 30 (030) and 120 will give a poor intersection. If both are multiplied by 7, the products are 210 and 840. These values will give the same result with a much better intersection.

APPENDIX III

EFFECTS OF SMALL ERRORS WHEN USING HORIZONTAL BASE POSITION FINDING OR TWO-STATION SPOTTING SYSTEM

SECTION I

HORIZONTAL BASE SYSTEM

■ 1. EFFECTS OF SMALL CHANGES IN OBSERVED AZIMUTHS.—*a.* It is sometimes desirable to know the effect, particularly in range, produced by small changes in the azimuths measured at the base end stations; that is, the effect of errors in reading the azimuth of a target. The equations given below have been derived by differential calculus for this purpose. Their similarity to the equations for range and lateral deviations used in spotting will be noted. (See ch. 7.)

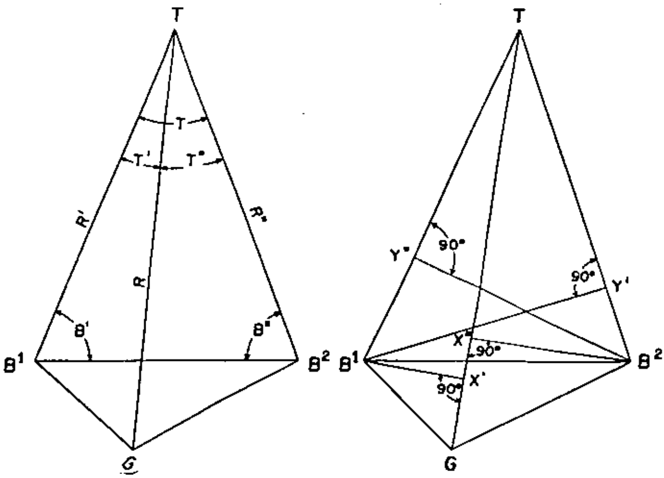


FIGURE 4.—Effect of small changes in observed azimuths, horizontal base.

$$\Delta R(\text{in yards}) = \pm\frac{R' \cos T''}{\sin T}\Delta B' \pm \frac{R'' \cos T'}{\sin T}\Delta B'' \qquad (1)$$

$$\Delta L(\text{in yards}) = \pm\frac{R' \sin T''}{\sin T}\Delta B' \pm \frac{R'' \sin T'}{\sin T}\Delta B'' \qquad (2)$$

In these equations, ΔR and ΔL are the effects in range and direction, respectively, from G of figure 4 due to the small changes in azimuth $\Delta B'$ and $\Delta B''$ from the observing stations B^1 and B^2, respectively. $\Delta B'$ and $\Delta B''$ must be in radians. If they are expressed in degrees, multiply by 0.01745.

The remaining terms of the equations are illustrated in figure 4. The signs of the right-hand terms depend on the direction in which the error $\Delta B'$ or $\Delta B''$ is made and must be determined by inspection. In the figure, an error to the right at B^1 gives a negative sign because its effect on the range is negative. By similar reasoning, an error to the right at B^2 gives a positive sign. The values for substitution in the equations may be read from a small scale plot of the situation; a scale of 1,000 yards to the inch should be suitable. The only precaution to be noted pertains to the measurement of the small angles T, T', and T''. Whenever the sines of these angles are used, the angles must be measured carefully to avoid large errors. If it is possible to determine the values of these angles by taking azimuth difference, these values should be taken in preference to values measured from the plot. Equations (1) and (2) may be expressed entirely in linear values as follows, in which case measurement of the angles is avoided (fig. 4):

$$\Delta R = \pm\frac{(R')(TX'')}{B''Y''}\Delta B' \pm \frac{(R'')(TX')}{B'Y'}\Delta B'' \qquad (3)$$

$$\Delta L = \pm\frac{(R')(B''X'')}{B''Y''}\Delta B' \pm \frac{(R'')(B'X')}{B'Y'}\Delta B'' \qquad (4)$$

b. For convenience, let the letters M and N represent the coefficients of $\Delta B'$ and $\Delta B''$, respectively. Then—

$$M_R = \frac{R' \cos T''}{\sin T} = \frac{(R')(TX'')}{B''Y''}$$

$$M_L = \frac{R' \sin T''}{\sin T} = \frac{(R')(B''X'')}{B''Y''}$$

$$N_R = \frac{R'' \cos T'}{\sin T} = \frac{(R'')(TX')}{B'Y'}$$

$$N_L = \frac{R'' \sin T'}{\sin T} = \frac{(R'')(B'X')}{B'Y'}$$

and

$$\Delta R = \pm M_R \Delta B' \pm N_R \Delta B''$$

$$\Delta L = \pm M_L \Delta B' \pm N_L \Delta B''$$

The positions of B^1, B^2, and G being fixed, then to each point T in the field of fire there corresponds a definite value of M and a definite value of N. An error of $\Delta B'$ in the measurement of the B^1 azimuth accompanied by a zero error in the measurement of the B^2 azimuth will result in an error of $M_R \Delta B'$ in the determination of the range, and of $M_L \Delta B'$ in the determination of the azimuth from the directing point to the target. The errors made at the two stations are independent of each other. If an error is made at each station, their results must be added algebraically (*a* above) to find their combined effect.

■ 2. PROBABLE ERROR.—*a*. The errors made by the base end observers may be considered to be normally distributed, that is, according to the curve of accidental errors. In artillery practice, the probable error r' of an observer in reading azimuths is taken as the product of his mean error times the factor 0.845. The corresponding probable error in range is equal to the probable error in azimuth times the factor M_R (or N_R). The probable error in direction may be found by a similar combination.

b. Since the errors at the two stations are normally distributed and independent of each other and since their results must be added algebraically to find their combined error, the combined result of the corresponding probable errors may be found by compounding those probable errors as follows:

The total range probable error

$$r_R = \sqrt{(M_R r')^2 + (N_R r'')^2}$$

The total lateral probable error

$$r_L = \sqrt{(M_L r')^2 + (N_L r'')^2}$$

NOTE.—A more complete discussion of probable errors is found in paragraphs 105, 129, 130, and 131.

■ 3. EXAMPLES.—*a.* Assume that the values in figure 4 are as follows:

$$T' = 1°45' \qquad R' = 15{,}000 \text{ yards}$$
$$T'' = 9°30' \qquad R'' = 18{,}000 \text{ yards}$$
$$T = 11°15'$$

Find the change in range corresponding to an error of 0.25° in the B^1 azimuth alone.

Solution:

$$M_R = \frac{15{,}000 \cos 9°30'}{\sin 11°15'}; \ \Delta B' = 0.25 \times 0.01745$$

$$\begin{aligned} \log 15{,}000 &= 4.17609 \\ \log \cos 9°30' &= 9.99400-10 \\ \text{colog} \sin 11°15' &= 0.70977 \\ \hline \log M_R &= 4.87986 \\ M_R &= 75{,}833 \end{aligned}$$

$$\begin{aligned} \log 0.25 &= 9.39794-10 \\ \log 0.01745 &= 8.24180-10 \\ \hline \log \Delta B' &= 7.63974-10 \\ \Delta B' &= 0.004362 \text{ radian} \end{aligned}$$

$$\begin{aligned} \Delta R = M_R \Delta B' &= 75{,}833 \times 0.004362 \\ \log 75{,}833 &= 4.87986 \\ \log 0.004362 &= 7.63974-10 \\ \hline \log \Delta R &= 2.51960 \\ \Delta R &= 331 \text{ yards} \end{aligned}$$

b. In the same situation, find the change in range corresponding to an error of 0.25° in the B^2 azimuth alone.

Solution:

$$N_R = \frac{18{,}000 \cos 1°45'}{\sin 11°15'}$$

$$\begin{aligned} \log 18{,}000 &= 4.25527 \\ \log \cos 1°45' &= 9.99980-10 \\ \text{colog} \sin 11°15' &= 0.70977 \\ \hline \log N_R &= 4.96484 \\ N_R &= 92{,}223 \\ \Delta B'' = \Delta B' &= 0.004362 \text{ radian} \\ \Delta R = N_R \Delta B'' &= 92{,}223 \times 0.004362 \\ \log 92{,}223 &= 4.96484 \\ \log 0.004362 &= 7.63974-10 \\ \hline \log \Delta R &= 2.60458 \\ \Delta R &= 402 \text{ yards} \end{aligned}$$

c. Recent experiments indicate that the probable error to be expected of an experienced enlisted observer using an accurately oriented D. P. F. or an azimuth instrument is about 0.005°. Applying this value to the situation in *a* above, find the probable error in range finding so far as the measurement of base-end azimuths is concerned.

Solution:

$$r_R = \sqrt{(M_R r')^2 + (N_R r'')^2}$$
$$M_R = 75{,}833 N_R = 92{,}223$$
$$r' = r'' = 0.005 \times 0.01745 \text{ radian} = 0.00008725 \text{ radian}$$

By slide rule—

$$(M_R r')^2 = 43.8$$
$$(N_R r'')^2 = 64.7$$
$$(r_R)^2 = 108.5 \text{ yards}$$
$$r_R = 10 \text{ yards}$$

d. The equation given above shows the probable error in range finding due to errors in measurement of azimuth alone and does not touch the problem of errors made in plotting these azimuths on the plotting board. The latter will depend upon the geometrical figure just as in the preceding discussion, and in addition it will depend upon the type of board, its mechanical condition, and the scale of plotting. If the probable error in indicated range due to operation of the plotting board becomes known, this value may be compounded with r_R of the preceding discussion by the rule of the square root of the sum of the squares. For example, assume that the probable error of the plotting board is equal to that of the observation as calculated in *c* above, and find the total probable error in range finding.

Solution:

$$r_R = \sqrt{(31)^2 + (31)^2} = 31 \times \sqrt{2}$$
$$= 31 \times 1.14 = 43 \text{ yards}$$

Note that this deals only with the accidental errors and not with the systematic errors, so that this value shows the magnitude of the irregularities to be expected in the plotted course.

SECTION II

TWO-STATION SPOTTING SYSTEM

■ 4. ERRORS.—Denoting the true values of the angular deviations of a splash from the target by $\Delta S'$ and $\Delta S''$ and the errors of the spotters in measuring those angles $\delta S'$ and $\delta S''$, and using the values of M_R and N_R as given in paragraph 1*b*, the indicated value of the range deviation would be

$$\begin{aligned}\Delta R' &= M_R(\Delta S' + \delta S') + N_R(\Delta S'' + \delta S'')\\ &= (M_R\Delta S' + N_R\Delta S'') + (M_R\delta S' + N_R\delta S'')\end{aligned}$$

While the true value of the deviation would be

$$\Delta R = M_R\Delta S' + N_R\Delta S''$$

The error in spotting, δR, would be

$$\delta R = \Delta R' - \Delta R = M_R\delta S' + N_R\delta S''$$

■ 5. EXAMPLE.—Given the values $T'=50$ mils, $T''=230$ mils, $T=280$ mils, $R'=14{,}800$ yards, and $R''=15{,}200$ yards. What range spotting error corresponds to an angular error of 1 mil at each station if both errors are in the same sense?

Solution:

$$\delta R = M_R\delta S' + N_R\delta S''$$

$\log R' = 4.17026$	$\log R'' = 4.18184$
$\log \cos T'' = 9.98884-10$	$\log \cos T' = 9.99947-10$
$\text{colog} \sin T = 0.56633$	$\text{colog} \sin T = 0.56633$
$\log M_R = 4.72543$	$\log N_R = 4.74764$
$\log 0.0009817 = 6.99198-10$	$\log 0.0009817 = 6.99198-10$
$\log \delta S' = 0.00000$	$\log \delta S'' = 0.00000$
$\log M_R\delta S' = 1.71741$	$\log N_R\delta S'' = 1.73962$
$M_R\delta S' = 52.2$ yards	$N_R\delta S'' = 54.9$ yards

$$\delta R = 107 \text{ yards}$$

■ 6. PROBABLE ERROR.—When the probable errors of the spotting observers are known, the probable error of spotting results may be computed just as the probable error of range finding was computed in paragraph 2. For example, if the spotting observers are using the azimuth instrument, M1918, the least reading of the interior splash scale is 5 mils and they must interpolate between these graduations. Under these conditions, it seems likely that the probable error of the reading would be 1 mil and the range probable error of the spotting system, exclusive of errors incidental to the operation of the spotting board, would be

$$r_R = \sqrt{(Mr')^2 + (Nr'')^2}$$
$$= \sqrt{(52.2)^2 + (54.9)^2}$$

$\log 52.2 = 1.71741$ $\qquad$ $\log 54.9 = 1.73962$

$\log (52.2)^2 = 3.43482$ $\qquad$ $\log (54.9)^2 = 3.47924$

$(52.2)^2 = 2{,}722$ $\qquad$ $(54.9)^2 = 3{,}015$

$(r_R)^2 = 2{,}722 + 3{,}015 = 5{,}737$

$\log (r_R)^2 = 3.75868$

$\log r_R = 1.87934$

$r_R = 76$ yards

APPENDIX IV

PRINCIPLES OF VERTICAL BASE POSITION FINDING

SECTION I

THEORETICAL PRINCIPLES

■ 1. CURVATURE OF THE EARTH.—*a*. In vertical base position finding, the range to the target is determined by the general equation tan α=height of instrument÷range, where α is the measured angle between the horizontal and a line from the instrument to the water line of the target. This determination is affected by curvature of the earth and atmospheric

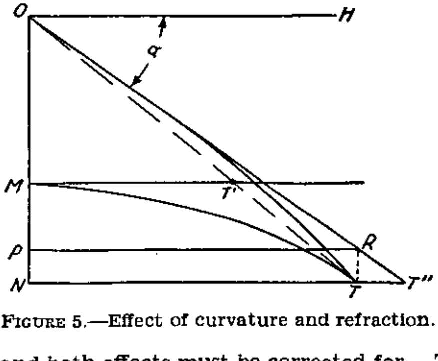


FIGURE 5.—Effect of curvature and refraction.

refraction and both effects must be corrected for. The effects are illustrated in figure 5. *MT* represents the surface of the earth, *T* a target, and *O* an observing instrument with a height of instrument *OM*. If no refraction were present, the target would appear on the line *OT* and a true height of instrument *OM* would give a range *MT'* whereas the desired range is *NT* which differs by a negligible quantity from the map range. In order to correct for curvature of the earth, therefore, the height of instrument used in thc computation must be increased by MN.

b. The value of MN depends on the radius of curvature of the earth's surface and the range. In figure 6, O, M, N, and T are shown as before, MA is a tangent, and C represents the center of the earth. From this figure—

$$AC^2=(AT+TC)^2=AT^2+2AT\times TC+TC^2$$
$$AC^2=MA^2+MC^2$$
$$\text{and } AT^2+2AT\times TC+TC^2=MA^2+MC^2$$

Since the angle subtended by the arc MT is relatively small, MT and MA are nearly equal and AT is relatively small. Also

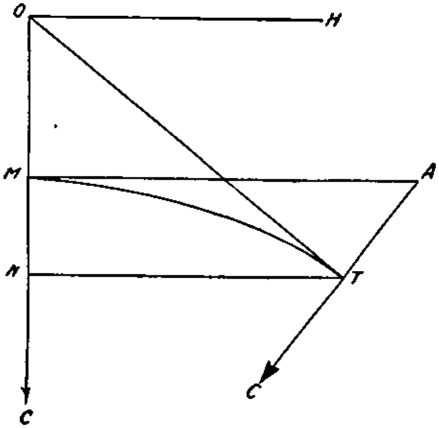


FIGURE 6.—Magnitude of curvature effect.

AT is nearly equal to MN, the effect of curvature. Canceling the equal terms TC^2 and MC^2, and dropping AT^2 as negligible

$$AT=\frac{MA^2}{2TC}$$

Let h_c represent the effect of curvature MN $(=AT)$, R the range MT $(=MA)$, and r the radius of the earth TC. The expression then becomes

$$h_c=\frac{R^2}{2r}$$

NOTE.—A mean value of r=6,963,455 yards has been used in the calculations of this appendix.

■ 2. ATMOSPHERIC REFRACTION.—The rays of light between an observer and a target are bent downward by refraction. As a result, the target in figure 5 will appear on the line ORT'' instead of OT. This would give a range NT'' whereas the desired range is still the range NT. Therefore the height of instrument used in the computation must be decreased by NP, giving a corrected height of instrument of OP. If the effect of refraction NP is represented by h_r,

$$h_r = m\frac{R^2}{r}$$

where m is the coefficient of refraction (par. 58*b*).

A value of $m = .0714$ may be used in the calculation of firing data.

■ 3. COMBINED EFFECT OF CURVATURE AND REFRACTION.—Since, in seacoast artillery work, the effect of refraction is always to make the apparent effect of curvature less than the true value, the combined expression for curvature and refraction becomes

$$h = h_c - h_r = (1-2m)R^2/2r$$

Table I, appendix VI, gives the vertical effect of curvature and refraction combined for ranges between 1,000 and 50,000 yards.

■ 4. RANGE FINDING BY THE DEPRESSION ANGLE.—*a*. The range problem that is solved by a depression position finder is as follows (fig. 5):

$$\tan HOR = \tan ORP = (OM + MP)/PR$$

Let α represent the depression angle HOR; b, the true height of instrument OM; R, the range to the target PR; and h, the combined vertical correction due to curvature and refraction MP. Then the equation may be written

$$\tan \alpha = (b+h)/R$$

Substituting for h its value $(1-2m)R^2/2r$

$$\tan \alpha = \frac{b}{R} + \frac{1-2m}{2r}R$$

Let $C = (1-2m)/2r$

Then

$$\tan \alpha = \frac{b}{R} + CR$$

Solving for R, we get

$$R = \frac{\tan \alpha - \sqrt{\tan^2 \alpha - 4Cb}}{2C}$$

b. The larger of the roots obtainable is not shown because it is of no value in this case. It should be noted that b, h, r, and R must be expressed in the same unit. The construction of an instrument to solve this equation for any given height of instrument and any given condition of refraction is not difficult, provided there is a suitable ratio between the height of the instrument and the maximum range to be measured. Since the tide changes the effective height of instrument continually and since it would be impracticable to make a different instrument for every height of station, instruments are designed so as to be adjustable within certain limits as to the height at which they are to work. The principle which is used in designing this feature of most of our instruments is discussed in "The Journal of the United States Artillery," 1909, volume 31, page 48. Instruments in our service are designed to correct automatically for the effects of curvature and a normal refraction of the ratio $\frac{1}{14}$ or ($m=0.0714$). This is a mean for all values of refraction.

c. A change in refraction produces an apparent change in the height of the target relative to the instrument. Correction for such varying refraction may be made by changing the setting of the height scale on the instrument to make it read the correct range to a datum point in the part of the field of fire in which it is expected that the instrument will be used. The correction may also be made by a mechanical change in the angle of depression corresponding to the range to the datum point without varying the height setting. A combination of the two methods is used in our service.

■ 5. Value of C for Use in Computations:

$$C=\frac{(1-2m)}{2r}=\text{the vertical effect of curvature}$$

and refraction when $R=1$ yard. Based upon the values $r=6{,}963{,}455$ yards and $m=0.0714$, the following values may be listed:

$$C=0.000000061550$$
$$\log C=2.78923-10$$
$$\log 2C=3.09026-10$$
$$\log 4C=3.39129-10$$

■ 6. EXAMPLE.—Given a height of instrument of 120 feet and a coefficient of refraction of 0.0714, what range corresponds to an angle of depression of 15 minutes?

Solution:

$$R=\frac{\tan \alpha-\sqrt{\tan^2 \alpha-4Cb}}{2C},$$

where $\alpha=15'$ and $b=120$ feet$=40$ yards

Term	Logarithm	Natural number	
$\tan \alpha$	7.63982−10		0.0043633
$\tan^2 \alpha$	5.27964−10	0.000019039	
$4c$	3.39129−10		
b	1.60206		
$4Cb$	4.99224−10	0.000009848	
$\tan^2 \alpha-4Cb$	4.96336−10	0.000009191	
$\sqrt{\tan^2 \alpha-4Cb}$	7.48168−10		0.0030317
Numerator	7.12437−10		0.0013316
Denominator ($2C$)	3.09026−10		
R	4.03411	10,817 yards.	

SECTION II

EFFECT OF SMALL CHANGES IN THE DEPRESSION ANGLE

■ 7. GENERAL.—The range effect of small changes in the depression angle may be determined by differentiating the equation

$$\tan \alpha=\frac{b}{R}+CR$$

The result of the operation is

$$\Delta R=-\frac{R^2}{b-h}\Delta\alpha-\frac{(b+h)^2}{b-h}\Delta\alpha$$

Due to its relatively small size the term $\frac{(b+h)^2}{b-h}\Delta\alpha$ may be ignored for practical purposes and the equation becomes

$$\Delta R = -\frac{R^2}{b-h}\Delta\alpha \tag{1}$$

in which the terms have the following values:

R=the range in yards.

b=the height of instrument in yards.

h=the vertical effect of curvature and refraction in yards.

ΔR=the change in range in yards corresponding to a change of $\Delta\alpha$ radians in the depression angle.

Since a radian equals 206,265 seconds of arc, in order to use $\Delta\alpha$ in seconds of arc it becomes necessary to divide by 206,265. Sufficient accuracy is attained by making the figure 206,000, and equation (1) becomes

$$\Delta R = -\frac{R^2 \times \Delta\alpha}{(b-h)\times 206{,}000} \tag{2}$$

■ 8. PROBABLE ERROR.—If the probable error of the observer is r' seconds of arc, the probable error of range determination is

$$r_R = \frac{R^2 \times r'}{(b-h)\times 206{,}000}$$

■ 9. EFFECT OF MAGNIFICATION OF INSTRUMENT ON OBSERVED ANGLES.—The effect of magnification is to make smaller angles distinguishable. The distinguishable angle is taken as varying inversely as the power of the optical system. Let $\Delta\alpha'$ represent the accuracy index of the observer (par. 63) and M the magnifying power of the observing instrument. Then equation (2) above may be written as follows:

$$\Delta R = -\frac{R^2 \times \Delta\alpha'}{(b-h)\times M \times 206{,}000} \tag{3}$$

■ 10. EXAMPLES.—*a*. Given a height of instrument of 120 feet, a coefficient of refraction of 0.0714, and a depression angle of 15 minutes (corresponding range is 10,817 yards (par. 6)), what is the effect on the range of an increase of 1 minute in the depression angle?

Solution:

$$\Delta R=-\frac{R^2\times\Delta\alpha}{(b-h)\times 206{,}000}$$

R=10,817 yards $b-h$=33 yards
b= 40 yards $\Delta\alpha$=60 seconds
h= 7 yards

By slide rule $\Delta R=-1{,}030$ yards.

NOTE.—The range effect for a corresponding decrease in the depression angle would be +1.030 yards.

b. Assuming that the probable error of the observer and instrument in *a* above is 7 seconds of arc, what is the probable error of the range determination?

Solution:

$$r_R=\frac{R^2\times r'}{(b-h)\times 206{,}000}$$

R=10,817 yards $b-h$=33 yards
$r'=7''$

By slide rule r_R=120 yards.

c. As a result of tests with a 25-power D. P. F. having a height of site of 120 feet, it has been determined that a certain observer develops a mean error of 100 yards in range when observing at a mean range of 10,000 yards. What is the corresponding accuracy index of this observer?

Solution:

$$\Delta\alpha'=\frac{\Delta R\times(b-h)\times M\times 206{,}000}{R^2}$$

R=10,000 yards $b-h$=34 yards
b= 40 yards ΔR=100 yards
h= 6 yards M=25

By slide rule $\Delta\alpha'$=175 seconds.

SECTION III

EFFECT OF SMALL CHANGES IN HEIGHT OF INSTRUMENT

■ 11. GENERAL.—The effect on the indicated range caused by small changes in the height of instrument *b* may be found by finding the difference in the depression angle using the two values of b successively in the equation

$$\tan \alpha = \frac{b}{R} + CR$$

and then substituting the value of $\Delta\alpha$ thus found in equation (2) of paragraph 7. However, this effect may be determined with sufficient accuracy for practical purposes by the principle of similar triangles. In figure 7 any change in the angle of depression α will move the apparent position of the target

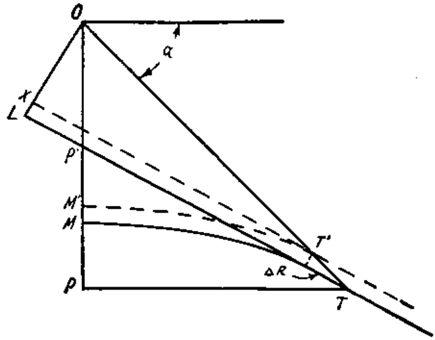


FIGURE 7.—Effect of small changes in height of instrument.

along the arc MT which represents the surface of the earth. The tangent LT may be considered as coinciding with MT in the vicinity of the target and therefore as the line along which the apparent position of the target will be moved. Since the angle PTP' is very small (as shown in figure 7 it is greatly exaggerated), MP' is approximately equal to MP, the difference between apparent and true level due to curvature and refraction, and OP' is approximately equal to $b-h$. This value is called the "effective height of instrument." In the right triangle OLT, LT is approximately equal to the range and OL to the effective height of instrument. Assume that after the tide correction was made the tide rose by a small amount XL. The instrument will indicate the range LT corresponding to the height OL, whereas the desired range is the range XT'. The error in range, ΔR, may be determined by the equation

$$\Delta R = LT \times XL/OL = R\Delta b/(b-h)$$

■ 12. EXAMPLE.—A D. P. F. has been adjusted to read the correct range by water lining on a datum point at 12,500 yards range. The height scale shows 150 feet as the height of instrument b. Assume that the tide has risen 1 foot since the adjustment was made. What range will be indicated on the instrument when it is again water lined on the datum point?

Solution:

$$\Delta R = R\Delta b/(b-h)$$

$R=$	12,500 yards	$h=$	29 feet
$\Delta b=$	1 foot	$b-h=$	121 feet
$b=$	150 feet		

By slide rule $\Delta R = 103$ yards.

$$R' = 12{,}500 + 103 \text{ yards} = 12{,}603 \text{ yards.}$$

NOTE.—Solution by equation (2) gives 12,613 yards.

APPENDIX V

EFFECTS OF SMALL ERRORS WHEN USING A SELF-CONTAINED RANGE FINDER

The geometric principles of a self-contained range finder are similar to those of a D. P. F. except that curvature and refraction are not involved. The basic equation is therefore simplified to

$$\tan \alpha = b/R.$$

In this case, b represents the length of the base line; that is, the distance between the penta prisms at each end of the range finder. α is called the parallax angle.

Differentiating this equation and transposing gives

$$\Delta R = -\frac{R^2}{b}\Delta\alpha \tag{1}$$

where $\Delta\alpha$ is expressed in radians.

Converting $\Delta\alpha$ from radians to seconds of arc the equation may be written

$$\Delta R = -\frac{R^2 \times \Delta\alpha}{b \times 206{,}000} \tag{2}$$

The form of equation containing the mean angular error of the observer is

$$\Delta R = -\frac{R^2 \times \Delta\alpha'}{b \times M \times 206{,}000} \tag{3}$$

These equations may be used in analyses of range finding in the same manner as those of Appendix IV.

APPENDIX VI

TABLES

TABLE I.—*Vertical effect of curvature and refraction* (par. 49 *c*)

$h=(1-2m)\frac{R^2}{2r}$, in which h, R, and r are expressed in the same unit

$C=\frac{(1-2m)}{2r}=0.0000000615499$ when $m=0.0714$ and $r=6{,}963{,}455$ yards ($\log C=2.78923-10$)

Range (thousands of yards)	Curvature and refraction (h) (in feet) $m=0.0714$	Range (thousands of yards)	Curvature and refraction (h) (in feet) $m=0.0714$	Range (thousands of yards)	Curvature and refraction (h) (in feet) $m=0.0714$
1	0.2				
2	.7	19	66.7	35	226.2
3	1.7	20	73.9	36	239.3
4	3.0	21	81.4	37	252.8
5	4.6	22	89.4	38	266.6
6	6.6	23	97.7	39	280.9
7	9.0	24	106.4	40	295.4
8	11.8	25	115.4	41	310.4
9	15.0	26	124.8	42	325.7
10	18.5	27	134.6	43	341.4
11	22.3	28	144.8	44	357.5
12	26.6	29	155.3	45	373.9
13	31.2	30	166.2	46	390.7
14	36.2	31	177.5	47	407.9
15	41.5	32	189.1	48	425.4
16	47.3	33	201.1	49	443.3
17	53.4	34	213.5	50	461.6
18	59.8				

NOTE.—Enter the table with R to the nearest 100 yards; take the effect to the nearest foot.

TABLE II–A.—*Factor—probability*

Factor	Probability	Factor	Probability	Factor	Probability	Factor	Probability
0.00	0.000	1.00	0.500	2.00	0.823	3.00	0.957
.05	.027	1.05	.521	2.05	.833	3.05	.960
.10	.054	1.10	.542	2.10	.843	3.10	.963
.15	.081	1.15	.562	2.15	.853	3.15	.966
.20	.107	1.20	.582	2.20	.862	3.20	.969
.25	.134	1.25	.601	2.25	.871	3.25	.972
.30	.160	1.30	.620	2.30	.879	3.30	.974
.35	.187	1.35	.638	2.35	.887	3.35	.976
.40	.213	1.40	.655	2.40	.895	3.40	.978
.45	.239	1.45	.672	2.45	.902	3.50	.982
.50	.264	1.50	.688	2.50	.908	3.60	.985
.55	.289	1.55	.704	2.55	.914	3.70	.987
.60	.314	1.60	.719	2.60	.920	3.80	.990
.65	.339	1.65	.734	2.65	.926	3.90	.992
.70	.363	1.70	.749	2.70	.931	4.00	.993
.75	.387	1.75	.762	2.75	.936	4.20	.995
.80	.411	1.80	.775	2.80	.941	4.40	.997
.85	.434	1.85	.788	2.85	.945	4.60	.998
.90	.456	1.90	.800	2.90	.949	4.80	.999
.95	.478	1.95	.812	2.95	.953	5.00	.999

TABLE II-B.—*Probability—factor*

Probability	Factor	Probability	Factor	Probability	Factor	Probability	Factor
0.01	0.019	0.26	0.492	0.51	1.024	0.76	1.742
.02	.037	.27	.512	.52	1.047	.77	1.780
.03	.056	.28	.532	.53	1.071	.78	1.819
.04	.074	.29	.551	.54	1.096	.79	1.858
.05	.093	.30	.571	.55	1.121	.80	1.900
.06	.112	.31	.592	.56	1.146	.81	1.943
.07	.130	.32	.612	.57	1.172	.82	1.988
.08	.148	.33	.632	.58	1.197	.83	2.035
.09	.167	.34	.652	.59	1.222	.84	2.084
.10	.186	.35	.673	.60	1.248	.85	2.134
.11	.205	.36	.693	.61	1.275	.86	2.185
.12	.224	.37	.714	.62	1.302	.87	2.239
.13	.243	.38	.735	.63	1.329	.88	2.301
.14	.262	.39	.757	.64	1.357	.89	2.365
.15	.281	.40	.778	.65	1.386	.90	2.439
.16	.299	.41	.800	.66	1.415	.91	2.514
.17	.318	.42	.822	.67	1.444	.92	2.596
.18	.337	.43	.843	.68	1.473	.93	2.687
.19	.357	.44	.864	.69	1.505	.94	2.788
.20	.376	.45	.886	.70	1.537	.95	2.906
.21	.395	.46	.909	.71	1.569	.96	3.044
.22	.415	.47	.931	.72	1.602	.97	3.218
.23	.434	.48	.954	.73	1.636	.98	3.451
.24	.453	.49	.977	.74	1.671	.99	3.815
.25	.473	.50	1.000	.75	1.706	1.00	

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